

24.400
Proseminar in philosophy I

Fall 2003

“Hardly anything more unfortunate can befall a scientific writer...”

$$(V) \quad \{x \mid Ax\} = \{x \mid Bx\} \leftrightarrow \forall x(Ax \leftrightarrow Bx)$$

This is a first-level version of Axiom V. To get an instance of (V), replace ‘ x ’ with an individual (first-order) variable, and replace ‘ Ax ’ by a formula with only that variable free, ditto for ‘ Bx ’. (V) can be split up into a good bit:

$$[V\odot] \quad \forall x(Ax \leftrightarrow Bx) \supset \{x \mid Ax\} = \{x \mid Bx\}$$

and a bad bit:

$$[V\ominus] \quad \{x \mid Ax\} = \{x \mid Bx\} \supset \forall x(Ax \leftrightarrow Bx)$$

Consider the formula ‘ $\exists F(x = \{y \mid Fy\} \ \& \ \sim Fx)$ ’, which expresses the concept *x is the extension of a concept that x does not fall under*. For convenience, let ‘ R ’ be a one-place predicate letter expressing this concept and let ‘ $\{R\}$ ’ denote its extension.

Does ‘ R ’ apply to $\{R\}$? If it does, then it doesn’t, and if it doesn’t, it does.

Contradiction.

More slowly and formally (with some obvious abbreviations). Suppose that $R\{R\}$;

i.e. for some G :

1. $\{R\} = \{G\} \ \& \ \sim G\{R\}$

By $[V\odot]$:

$$2. \{R\} = \{G\} \supset \forall x(Rx \leftrightarrow Gx)$$

By 1 and 2:

$$3. \forall x(Rx \leftrightarrow Gx)$$

By 1:

$$4. \sim G\{R\}$$

By 3 and 4:

$$\sim R\{R\}$$

So, by reductio:

$$6. \sim R\{R\}$$

Suppose that $\sim R\{R\}$; i.e. for every G:

$$7. \{R\} \neq \{G\} \vee G\{R\}$$

So:

$$8. \{R\} = \{G\} \supset G\{R\}$$

In particular:

$$9. \{R\} = \{R\} \supset R\{R\}$$

So:

$$10. R\{R\}$$

So, by reductio:

$$11. R\{R\}$$

By 6 and 11:

12. $\sim R\{R\} \ \& \ R\{R\}$ ■