Chapter Five
Ordering

We see that the semantics for modals (and conditionals) needs to make reference to an ordering among accessible worlds.

Contents

We have stressed throughout the previous two chapters that there are numerous parallels between quantification over ordinary individuals via determiner quantifiers and quantification over possible worlds via modal operators (including conditionals). Now, we turn to a phenomenon that (at least at first glance) appears to show that there are non-parallels as well: a sensitivity to an ORDERING of the elements in the domain of quantification. We first look at this in the context of simple modal sentences and later we'll look at conditionals.

5.1 The Driveway

Consider a typical use of a sentence like (98).

(98) John must pay a fine.

This is naturally understood in such a way that its truth depends both on facts about the law and facts about what John has done. For instance, it will be judged true if (i) the law states that driveway obstructors are fined, and (ii) John has obstructed a driveway. It may be false either because the law is different or because John's behavior was different.

What accessibility relation provides the implicit restriction of the quantifier *must* on this reading of (98)? A naïve attempt might go like this:

(99) λ*w*. λ*w*′. [what happened in *w*′ up to now is the same as what happened in *w*, and *w*′ conforms to what the law in *w* demands].

The problem with (99) is that, unless there were no infractions of the law at all in *w* up to now, no world *w*′ will be accessible from *w*. Therefore, (98) is predicted to follow logically from the premise that John broke some law. This does not represent our intuition about its truth conditions.

A better definition of the appropriate accessibility relation has to be more complicated:

(100) λ*w*. λ*w*′. [what happened in *w*′ up to now is the same as what happened in *w*, and *w*′ conforms at least as well to what the law in *w* demands as does any other world in which what happened up to now is the same as in *w*].

(100) makes explicit that there is an important difference between the ways in which facts about John's behavior on the one hand, and facts about the law on the other, enter into the truth conditions of sentences like (98). Worlds in which John didn't do what he did are simply excluded from the domain of *must* here. Worlds in which the law isn't obeyed are not absolutely excluded. Rather, we restrict the domain to those worlds in which the law is obeyed as well as it can be, considering what has happened. We exclude only those worlds in which there are infractions above and beyond those that are shared by all the worlds in which John has done what he has done. The analysis of (98) thus crucially involves the notion of an ordering of worlds: here they are ordered according to how well they conform to what the law in *w* demands.

5.2 Kratzer's Solution: Doubly Relative Modality

Kratzer proposes that modal operators are sensitive to *two* context-dependent parameters: a set of accessible worlds (provided by an accessibility function computed from a conversational background, the MODAL BASE), and a partial ordering of the accessible worlds (computed from another conversational background, called the ORDERING SOURCE).

Let's see how the analysis applies to the previous example.

- The modal base will be a function that assigns to any evaluation world a set of propositions describing the relevant circumstances, for example, what John did. Since in our stipulated evaluation world John obstructed a driveway, the modal base will assign the proposition that John obstructed

a driveway to this world. The set of worlds accessible from the evaluation world will thus only contain worlds where John obstructed a driveway.

- The ordering source will be a function that assigns to any evaluation world a set of propositions $\mathcal{P}$ whose truth is demanded by the law. Imagine that for our evaluation world this set of propositions contains (among others) the following two propositions: (i) nobody obstructs any driveways, (ii) anybody who obstructs a driveway pays a fine.
- The idea is now that such a set $\mathcal{P}$ of propositions can be used to order the worlds in the modal base. For any pair of worlds w_1 and w_2, we say that w_1 comes closer than w_2 to the ideal set up by $\mathcal{P}$ (in symbols: $w_1 <_{\mathcal{P}} w_2$), iff the set of propositions from $\mathcal{P}$ that are true in w_2 is a proper subset of the set of propositions from $\mathcal{P}$ that are true in w_1.
- For our simple example then, any world in modal base where John pays a fine will count as better than an otherwise similar world where he doesn't.
- Modals then make quantificational claims about the best worlds in the modal base (those for which there isn't a world that is better than them).
- In our case, (98) claims that in the best worlds (among those where John obstructed a driveway), he pays a fine.

More technically:

(101) Given a set of worlds X and a set of propositions $\mathcal{P}$, define the STRICT PARTIAL ORDER $<_P$ as follows:
$\forall w_1, w_2 \in X : w_1 <_{\mathcal{P}} w_2$ iff $\{p \in \mathcal{P} : p(w_2) = 1\} \subset \{p \in \mathcal{P} : p(w_1) = 1\}$.

(102) For a given strict partial order $<_{\mathcal{P}}$ on worlds, define the selection function $\max_{\mathcal{P}}$ that selects the set of $<_{\mathcal{P}}$-best worlds from any set X of worlds:
$\forall X \subseteq W : \max_{\mathcal{P}}(X) = \{w \in X : \neg\exists w' \in X : w' <_{\mathcal{P}} w\}$.

(103) $[\![\text{must}]\!]^{w,g} = \lambda f_{\langle s,st\rangle}.\ \lambda g_{\langle s,st\rangle}.\ \lambda q_{\langle s,t\rangle}.$
$\forall w' \in \max_{g(w)}(\cap f(w)) : q(w') = 1.$

TECHNICAL NOTE: This[1] only works if we can in general assume that the $<_P$ relation has minimal elements, that there always are accessible worlds that come closest to the P-ideal, worlds that are better than any world they can be compared with via $<_P$. It is possible, with some imagination, to cook up scenarios where this assumption fails. This problem has been discussed primarily in the area of the semantics of conditionals. There, Lewis presents relevant scenarios and argues that one shouldn't make this assumption, which he calls the Limit Assumption. Stalnaker, on the one other hand, defends the assumption against Lewis' arguments by saying that in actual practice, in actual natural language

1 For discussion see: Lewis [51], Stalnaker [74: Chapter 7, esp. pp. 140-142]. Further arguments against the Limit Assumption: Herzberger [33], Pollock [66]. Further arguments for the Limit Assumption: Warmbrod [80].

semantics and in actual modal/conditional reasoning, the assumption is eminently reasonable. Kratzer is persuaded by Lewis' evidence and does not make the Limit Assumption; hence her semantics for modals is more convoluted than what we have in (102) and (103). I will side with Stalnaker, not the least because it makes life easier.

EXERCISE 5.1: In her handbook article [48], Kratzer presents a number of examples of modal statements and sketches an analyses in terms of doubly relative modality. You should study her examples carefully. □

5.3 The Paradox of the Good Samaritan

Prior [70] introduced the following "Paradox of the Good Samaritan". Imagine that someone has been robbed and John is walking by. It is easy to conceive of a code of ethics that would make the following sentence true:

(104) John ought to help the person who was robbed.

In our previous one-factor semantics for modals, we would have said that (104) says that in all of the deontically accessible worlds (those compatible with the code of ethics) John helps the person who was robbed. Prior's point was that under such a semantics, something rather unfortunate holds. Notice that in all of the worlds where John helps the person who was robbed, someone was robbed in the first place. Therefore, it will be true that in all of the deontically accessible worlds, someone was robbed. Thus, (104) will entail:

(105) It ought to be the case that someone was robbed.

It clearly would be good not make such a prediction.

The doubly-relative analysis of modality can successfully avoid this unfortunate prediction. We conceive of (104) as being uttered with respect to a circumstantial modal base that includes the fact that someone was robbed. Among those already somewhat ethically deficient worlds, the relatively best ones are all worlds where John helps the victim.

Now, at first, it seems we still have that among the worlds in the modal base, all are worlds where someone was robbed, and we would thus appear to still make the prediction that (105) should be true. But this can now be fixed. For example, we could say that *ought* p is semantically defective if p is true throughout the worlds in the modal base. This could be a presupposition or some other ingredient of meaning. So, with respect to a modal base which pre-determines that someone was robbed, one couldn't felicitously say (105).

Consequently, saying (105) would only be felicitous if a different modal base is intended, one that contains both p and non-p worlds. And given a choice between worlds where someone was robbed and worlds where nobody was robbed,

most deontic ordering sources would probably choose the no-robbery worlds, which would make (105) false, as desired.

5.4 Kratzer's Version of the Samaritan Paradox

[to be written – see Kratzer's Handbook article]

5.5 Non-Monotonicity of Conditionals

The crucial role of an ordering of worlds in modal semantics also surfaces in the semantics of conditionals, as we would of course expect under the analysis of *if*-clauses as restrictors of modal operators. In this arena, the discussion usually revolves around the failure of certain inference patterns, which one would expect a universal quantifier to validate. Here are the most important ones:

(106) Left Downward Monotonicity ("Downward Entailingness")
Every A is a B. → Every A & C is a B.

(107) Transitivity
Every A is a B. Every B is a C. → Every A is a C.

(108) Contraposition
Every A is a B. → Every non-B is a non-A.

Conditionals were once thought to obey these patterns as well, known in conditional logic as Strengthening the Antecedent, Hypothetical Syllogism, and Contraposition. But then spectacular counterexamples became known through the work of Stalnaker and Lewis.

(109) Failure of Strengthening the Antecedent
a. If I strike this match, it will light.
If I dip this match into water and strike it, it will light.
b. If John stole the earrings, he must go to jail.
If John stole the earrings and then shot himself, he must go to jail.
c. If kangaroos had no tails, they would topple over. If kangaroos had no tails but used crutches, they would topple over.

(110) Failure of the Hypothetical Syllogism (Transitivity)
a. If Brown wins the election, Smith will retire to private life.
If Smith dies before the election, Brown will win the election.
If Smith dies before the election,Smith will retire to private life.
b. If Hoover had been a Communist, he would have been a traitor.
If Hoover had been born in Russia, he would have been a Com-

munist.
If Hoover had been born in Russia, he would have been a traitor.

(111) FAILURE OF CONTRAPOSITION

a. If it rained, it didn't rain hard.
If it rained hard, it didn't rain.

b. (Even) if Goethe hadn't died in 1832, he would still be dead now.
If Goethe were alive now, he would have died in 1832.

The Goethe example is due to Kratzer.

Note that there are examples of both "indicative" (epistemic) conditionals and counterfactual conditionals. It is sometimes thought that indicative conditionals are immune from these kinds of counterexamples, but it is clear that they are not. Also note that in (109b) we have a case of Failure of Strengthening the Antecedent with a deontic conditional. Deontic counterexamples to the other patterns seem harder to find.

The failure of these inference patterns indicates that the semantics of modal operators (restricted by *if*-clauses) is more complicated than the simple universal quantification we have been assuming. The intuitive diagnosis in all the trouble cases is that during the course of the inference, the modal quantifiers are suddenly quantifying over worlds that were not in the domain of quantification in the earlier steps.

The basic idea of most approaches to this problem is this: the semantics of conditionals is more complicated than simple universal quantification. The conditional does not make a claim about simply every antecedent world, nor even about every contextually relevant antecedent world. Instead, in each of the conditional statements, only a particular subset of the antecedent worlds is quantified over. Informally, we can call those the "most highly ranked antecedent worlds". Consider:

(112) If I had struck this match, it would have lit.
If I had dipped this match into water and struck it, it would have lit.

According to the Stalnaker-Lewis account, this inference is semantically invalid. The premise merely claims that the most highly ranked worlds in which I strike this match are such that it lights. No claim is made about the most highly ranked worlds in which I first dip this match into water and then strike it. Strengthening the Antecedent will only be safe if it is additionally known that the strengthened antecedent is instantiated among the worlds that verify the original antecedent.

The other fallacies receive similar treatments. Transitivity (Hypothetical Syllogism) fails for the new non-monotonic quantifier because even if all the most highly rated p-worlds are q-worlds and all the most highly rated q-worlds are r-worlds, we are not necessarily speaking about the same q-worlds (the q-worlds that p takes us to may be rather remote ones). So in the Hoover-example,

we get the following picture: The most highly ranked p-worlds in which Hoover was born in Russia (but where he retains his level of civic involvement), are all q-worlds in which he becomes a Communist. On the other hand, the most highly ranked q-worlds in which he is a Communist (but retaining his having been born in the United States and being a high level administrator) are all r-worlds in which he is a traitor. However, the most highly ranked p-worlds do not get us to the most highly ranked q-worlds, so the Transitive inference does not go through.

Contraposition fails because the fact that the most highly rated p-worlds are q-worlds does not preclude a situation where the most highly rated non q-worlds are also p-worlds. The most highly rated p-worlds in which Goethe didn't die in 1832 are all q-worlds where he dies nevertheless (well) before the present. But of course, the most highly rated (in fact, all) non-q-worlds (where he is alive today) are also p-worlds where he didn't die in 1832.

[much more to be written]

Supplementary Readings

The central readings for this chapter are two papers by Kratzer:

Kratzer, Angelika: 1991. "Modality." In Arnim von Stechow & Dieter Wunderlich (Editors) *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung*, pages 639–650. Walter de Gruyter Berlin.

Kratzer, Angelika: 1981. "The Notional Category of Modality." In H. J. Eikmeyer & H. Rieser (Editors) *Words, Worlds, and Contexts. New Approaches in Word Semantics*, pages 38–74. de Gruyter.

Some work that discusses and uses Kratzer's two factor semantics for modals:

Frank, Anette: 1996. *Context Dependence in Modal Constructions*. Ph.D. thesis, Universität Stuttgart. URL http://www.dfki.de/~frank/papers/header.ps.gz.

von Fintel, Kai & Iatridou, Sabine: 2004. "What to Do If You Want to Go to Harlem: Notes on Anankastic Conditionals and Related Matters." http://web.mit.edu/fintel/www/harlem.pdf.

Some work of mine that discusses whether non-monotonicity might have to be relegated to a dynamic pragmatic component of meaning:

von Fintel, Kai: 2001. "Counterfactuals in a Dynamic Context." In Michael Kenstowicz (Editor) *Ken Hale: A Life in Language*, pages 123–152. MIT Press Cambridge, MA. Preprint version: http://web.mit.edu/fintel/www/conditional.pdf.

VON FINTEL, Kai: 1999. "NPI Licensing, Strawson Entailment, and Context Dependency." *Journal of Semantics*, 16(2): 97–148. URL http://www3.oup.co.uk/semant/hdb/Volume_16/Issue_02/160097.sgm.abs.html. Preprint: http://web.mit.edu/fintel/www/npi.pdf.

[More references to come.]