

3.044 Problem Set 7

Deformation Processing, Evaporation

Due Friday April 29, 2005

1. Extrusion of a tube

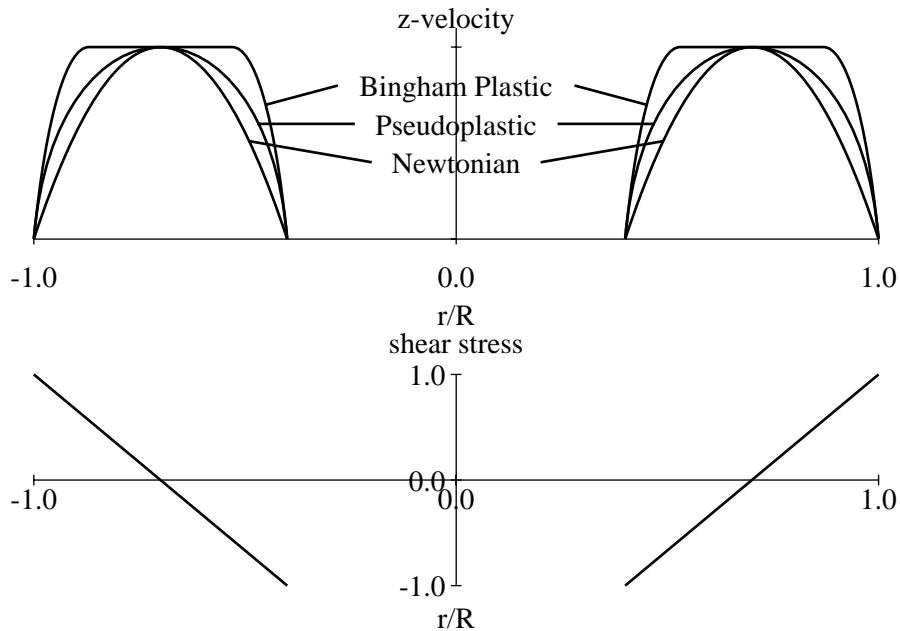
- (a) For all three of these flow behaviors, we have the same stress profile, which is the same as the flux profile for cylindrical (thermal) diffusion with generation:

$$\tau_{rz} = -\frac{1}{2} \frac{\partial p}{\partial z} r + \frac{A}{r}$$

(where $\partial p/\partial z$ is negative; you were not asked to derive this, but there it is). In the limit of a very thin annulus (so the plug nearly fills the tube), this is roughly linear like the stress distribution between parallel plates; we can use this simplified case to do the graphs.

- i. For a Newtonian fluid, this gives a parabolic flow profile.
- ii. For a pseudoplastic (shear-thinning) fluid, the magnitude of the shear stress is greatest near the walls, so the apparent viscosity is lowest there and highest in the center. This makes the velocity distribution flatter in the center, and more curved near the walls. (For example, for a power law pseudoplastic fluid with $n = 0.5$, the velocity distribution goes as $1 - y^3$ instead of the $1 - y^2$ parabolic Newtonian profile.)
- iii. For a Bingham plastic, the lower shear stress near the center will lead to a region with $\partial u_z/\partial r = 0$, so the velocity profile is flat there. If the maximum shear stress is above the yield stress τ_0 , then there will be a velocity gradient between this central solid region and the walls; otherwise the solid won't budge (unless there's some slip against the walls).

You were asked to graph these with the same maximum velocity, which should have looked like:



Shear stress τ_{rz} is positive on the outside because there z -momentum generated uniformly by the pressure gradient is diffusing in the positive- r direction; on the inside, that momentum is diffusing in the negative r -direction, so τ_{rz} is negative.

- (b) Solid aluminum deforms significantly as it flows past the plug; the resulting strain is likely to strengthen it quite a bit. Stronger means the yield stress will be high, so the center will likely flow as a solid (uniform velocity) as in the Bingham plastic model.

Needless to say, the simple Bingham plastic model does not capture the change in yield stress as the material flows through the extruder.

- (c) During extrusion, grains will be elongated in the direction of flow, and dislocation density will rise considerably, possibly resulting in formation of new low-angle grain boundaries. This corresponds to stage II or III deformation, where both the higher dislocation density and narrower grains result in higher yield stress.

[Not necessary for full credit] If the temperature is high enough, recrystallization could dynamically form new grains with low defect densities, which would then be sheared along with the rest of the material. This would reduce the force required to drive material through the extruder, and likely produce less strengthening than at lower temperatures.

2. Sintering rates and stages

- (a) There is no densification associated with the evaporation/condensation sintering mechanism, because densification requires that material be moved from between the grains (*i.e.* from the grain boundary) to the neck, whereas this mechanism simply moves material from the exposed grain surface to the neck.
- (b) To determine the relationship between the time required to reach a given x/r ratio when sintering $50\mu\text{m}$ CdCl₂ powder vs. $100\mu\text{m}$ powder, we first solve the equation describing neck radius for the time, and then take the ratio of the two times:

$$\frac{x}{r} = \left(\frac{\sqrt{2}\gamma p_{v0}}{3d^2\sqrt{\pi MR^3 T^3}} \right)^{\frac{1}{3}} r^{-\frac{2}{3}} t^{\frac{1}{3}}$$

$$t = \left(\frac{x}{r} \right)^3 r^2 \frac{3d^2\sqrt{\pi MR^3 T^3}}{\sqrt{2}\gamma p_{v0}}$$

$$\frac{t_{50\mu\text{m}}}{t_{100\mu\text{m}}} = \frac{\left(\frac{x}{r} \Big|_{50\mu\text{m}} \right)^3 (25\mu\text{m})^2 \frac{3d^2\sqrt{\pi MR^3 T^3}}{\sqrt{2}\gamma p_{v0}} \Big|_{50\mu\text{m}}}{\left(\frac{x}{r} \Big|_{100\mu\text{m}} \right)^3 (50\mu\text{m})^2 \frac{3d^2\sqrt{\pi MR^3 T^3}}{\sqrt{2}\gamma p_{v0}} \Big|_{100\mu\text{m}}}$$

Since d , M , R , T , p_{v0} and the ratio x/r at the end of stage I are the same for both powder sizes, this simplifies to:

$$\frac{t_{50\mu\text{m}}}{t_{100\mu\text{m}}} = \frac{(25\mu\text{m})^2}{(50\mu\text{m})^2} = \frac{1}{4},$$

so if the $100\mu\text{m}$ powder starts rapid densification in ten hours, the $50\mu\text{m}$ powder starts it in two and a half.

3. Evaporation of alloying elements from titanium

- (a) See the results in the table below.
- (b) This used the equation for the rate constant of a dilute solute B in solvent A derived in class:

$$k''_B = \frac{M_A}{\rho_A} \frac{\gamma_B \bar{p}_{vB}}{\sqrt{2\pi M_B R T}} = \frac{M_A}{\rho_A} \gamma_B J_{evB}$$

Note that these (and vapor pressures and evaporation rates) are *much* lower than the corresponding values for Cd, Pb and Zn calculated in the previous problem set.

- (c) Whether solutes increase or decrease in concentration depends on whether they evaporates less or more than the solvent relative to their concentration. This in turn is given by the evaporation ratio:

$$ER_B = \frac{\text{wt}\%_{B,\text{vapor}}/\text{wt}\%_{A,\text{vapor}}}{\text{wt}\%_{B,\text{liquid}}/\text{wt}\%_{A,\text{liquid}}}$$

If greater than one, the solute evaporation rate is proportionally faster than the solvent, and it will decrease in concentration; the opposite is also true. This ratio can be calculated from thermodynamic and physical properties:

$$ER_B = \gamma_B \frac{\bar{p}_{vB}}{\bar{p}_{vA}} \sqrt{\frac{M_A}{M_B}}$$

Results are summarized in the table below, and show that aluminum, with an evaporation (much) greater than one, will decrease in concentration, while vanadium, with evaporation ratio less than one, will increase in concentration.

Calculation results:

Element	\bar{p}_v , torr	J_{ev} , $\frac{\text{mol}}{\text{m}^2 \cdot \text{s}}$	k'' , $\frac{\text{m}}{\text{s}}$	ER
Al	6.34	15.8	1.18×10^{-5}	43.9
Ti	0.0123	0.0326	—	—
V	2.39×10^{-3}	4.34×10^{-3}	5.07×10^{-8}	0.188

- (d) Intense heating on a small spot, resulting in intense local evaporation, will produce “focusing” in the vapor plume, *i.e.* the flux upward from the center will be considerably higher than that predicted by the cosine distribution, and that on the sides considerably lower. This is often expressed as a cosine power distribution: $J \propto \cos^n \theta$, where n observed in industrial evaporators can be from two or three to as high as four or five.
- (e) The deposited metal will have roughly the same composition as the vapor. Thus, we can use the evaporation ratios calculated above to determine the ratio of elements needed in the liquid to evaporate at the correct vapor composition:

$$\frac{\text{wt}\%_{B,\text{liquid}}}{\text{wt}\%_{A,\text{liquid}}} = \frac{\text{wt}\%_{B,\text{vapor}}/\text{wt}\%_{A,\text{vapor}}}{ER_B}$$

For aluminum and vanadium:

$$\frac{\text{wt}\%_{\text{Al},\text{liquid}}}{\text{wt}\%_{\text{Ti},\text{liquid}}} = \frac{6\text{wt}\%/90\text{wt}\%}{43.9} = 1.52 \times 10^{-3}$$

$$\frac{\text{wt}\%_{\text{V},\text{liquid}}}{\text{wt}\%_{\text{Ti},\text{liquid}}} = \frac{4\text{wt}\%/90\text{wt}\%}{0.188} = 0.236$$

These ratios give a liquid composition which is 0.123wt% aluminum, 81wt% titanium, and 19wt% vanadium.

Note: if vanadium were not so close to ideal behavior in titanium solution, such a high fraction would likely violate Henry’s law for linearity of activity.