2.6 February 18, 2005: Convective cooling curves

Result from last time, dimensionless numbers and the equation:

$$\frac{T - T_{fl}}{T_i - T_{fl}} = f\left(\frac{r}{R}, \frac{\alpha t}{R^2}, \frac{hR}{k}\right). \tag{2.41}$$

Note on dimensional analysis and ambiguity of choice on parameters to keep, eliminate, with example of one layer and heat transfer coefficient. Keeping q_x and h is just more elegant than q_x and k, though the dimensional analysis allows either one.

Now can graph π_T vs. π_r for various π_t, different graphs for different π_h. Large (> 100) reverts to the constant temperature boundary condition $T = T_{fl}$, small (< 0.1) we'll get to in a moment, intermediate Biot number graphs.

The handout (P&G pp. 292–299) takes a different approach to the graphs: π_T vs. π_t for various π_h, graphs at different π_r. Useful for temperature histories like PS2#4 (but skip past the early graphs...), and also for TTT diagrams, like our ceramic thermal spray. But not so useful for uniformity. P. 299 has what we did.

Newtonian cooling Small Biot number (< 0.1): temperture is roughly uniform. Let's say it *is* uniform. Then we just have $T(t)$, $\pi_T(\pi_t, \pi_h)$. Cool.

Balance over the entire object: accumulation = -out.

$$V\frac{dH}{dt} = -Aq_r \tag{2.42}$$

$$V\rho c_p\frac{dT}{dt} = -Ah(T - T_{fl}) \tag{2.43}$$

Rearrange:

$$\frac{dT}{T - T_{fl}} = -\frac{Ah}{V\rho c_p}dt \tag{2.44}$$

Integrate, with initial condition T_i at $t = 0$:

$$\ln(T - T_{fl}) - \ln(T_i - T_{fl}) = -\frac{Aht}{V\rho c_p} \tag{2.45}$$

$$\frac{T - T_{fl}}{T_i - T_{fl}} = \exp\left(-\frac{Aht}{V\rho c_p}\right) \tag{2.46}$$

So far, everything's general, with volume and area, so whether a sphere, rod, plate, or crumpled up piece of paper, it just works.

First, examine terms, timescale, larger/smaller h, rho c_p, V/A. Plug in V/A:

- Sphere: $R/3$
- Cylinder: $R/2$
- Plate: "R" $= L/2$
- Other shapes: varies...

Can instead define alternate Biot and Fourier numbers: $\text{Bi}' = \frac{hV}{kA}$, $\text{Fo}' = \frac{\alpha A^2}{V^2}t$, then:

$$\frac{T - T_{fl}}{T_i - T_{fl}} = \exp\left(-\frac{hV}{kA}\frac{kA^2}{\rho c_p V^2}t\right) = \exp\left(-\text{Bi}'\text{Fo}'\right). \tag{2.47}$$

So, all set for PS2?

Thermal conductivity Diffusion is straightforward: atoms move, right? Well, not quite: gases in straight lines, liquid atoms move in chains, vacanices, interstitials, dislocations, etc. For heat, various mechanisms:

- Collisions
- Phonons
- Photons—radiation, which is spontaneous emission from hot body
- Electrons

On electrons, Wiedmann-Franz law:

$$k_{el} = L\sigma_{el}T, L = \frac{\pi}{3}\left(k_B/e\right)^2 = 2.45\times 10^{-8}\frac{\text{Wohm}}{K^2}$$

where e=electron charge.

Metals: σ_{el} goes down with temperature. What about electrons is semiconductors?

Liquids: water .615 20-100°C, O_2 3.4×10^{-4}, H_2 1.77×10^{-3} (both 300K)

Influence of porosity and humidity/water absorption. Gases are very bad conductors, water not quite as bad but has very high specific heat! (PS2 #1d, water has four times c_p of aluminum which is highest there.)

Typical conductivity values: 0.1 to 300 $\frac{\text{W}}{\text{m}\cdot\text{K}}$. Porous→less, metals high, gases *really* small!

Note: at conference, diamond-aluminum composite for microelectronics, 45 vol% diamond but isotropic conductivity of 550 W/mK! Nearly twice copper, squeeze-castable into heat sink parts. Q: why no diamond-iron composite?