1. Dimensional analysis: diffusion of a zinc coating

   The "Shrinking Gaussian" solution to the time-dependent diffusion equation:

$$C = \frac{\beta}{\sqrt{\pi D t}} \exp\left(-\frac{x^2}{4Dt}\right)$$

   (a) Dimensions and units:

Dimension	Units
C	$\frac{\text{mol}}{\text{m}^3}$
β	$\frac{\text{mol}}{\text{m}^2}$
D	$\frac{\text{m}^2}{\text{s}}$
t	seconds
x	meters

   Note: substitution of g or kg for mol works just as well.

   (b) As we can see above, there are five dimensions, and three base units (mol or g or kg, m, s), therefore, there are just two dimensionless parameters.

   (c) We want to construct π_C and π_x, eliminating β, D and t.

Dimension	mol	m	s
C	1	-3	0
β^{-1}	-1	2	0
$D^{1/2}$	0	1	$-\frac{1}{2}$
$t^{1/2}$	0	0	$\frac{1}{2}$
Total	0	0	0

Dimension	mol	m	s
x	0	1	0
β^0	0	0	0
$D^{-1/2}$	0	-1	$\frac{1}{2}$
$t^{-1/2}$	0	0	$-\frac{1}{2}$
Total	0	0	0

   So we have:

$$\pi_C = \frac{C\sqrt{Dt}}{\beta}$$

$$\pi_x = \frac{x}{\sqrt{Dt}}$$

   (d) Start with the solution itself:

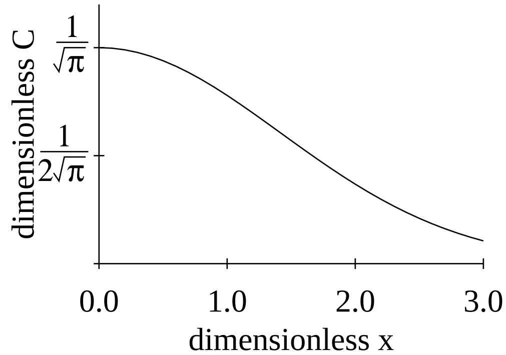
$$C = \frac{\beta}{\sqrt{\pi D t}} \exp\left(-\frac{x^2}{4Dt}\right)$$

   One rearrangement should do it:

$$\frac{C\sqrt{Dt}}{\beta} = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4Dt}\right)$$

$$\pi_C = \frac{1}{\sqrt{\pi}} \exp\left(-\frac{\pi_x^2}{4}\right)$$

   (e) When made dimensionless like this, the "Shrinking Gaussian" becomes a simple Gaussian with a maximum value of $1/\sqrt{\pi}$:



As for the width, you could define any measure of it, and give the corresponding value. One such measure is where the Gaussian reaches $1/e$ of its maximum; this is relatively easy because you just set the exponent to -1, so $-\pi_x^2/4 = -1$ and $\pi_x = 2$.

Another popular measure for Gaussian distributions is the "full width at half maximum" (FWHM). For this, you solve:

$$\exp\left(-\frac{\pi_x^2}{4}\right) = \frac{1}{2}$$

$$\pi_x = 2\sqrt{\ln 2} \simeq 1.39$$

Either of these measures was fine.