Viscous Flow, Drag Force

3.044 March 18, 2005

Mechanics:
- Problem Set 4 due March 30

Today's lecture:
- Terminal rising/sinking velocity
- Reynolds number and stability
- Engineering and Society: Lessons from 9/11

Inclusions and Bubbles: Drag Force

Roughly speaking: sum of viscous drag and kinetic energy terms

Sphere:

$$F_d = 3\pi d\eta V + 0.44 \cdot \frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2$$

Bubble:

$$F_d = 2\pi d\eta V$$

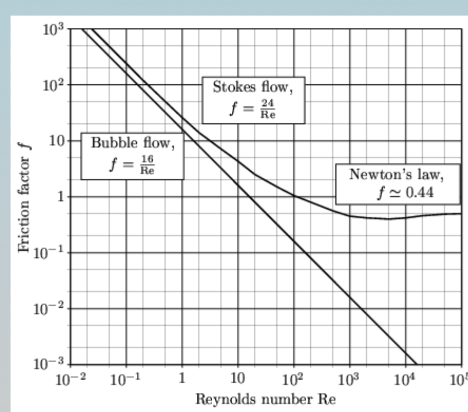
Ratio of kinetic energy to viscous contribution: Reynolds number

$$\mathrm{Re} = \frac{\rho V d}{\eta}$$

General friction factor:

$$F_d = fKA = f \cdot \frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2$$

Solid extremes: $f = \frac{24}{\mathrm{Re}} \Rightarrow f = c_d$



Calculating Velocity

Bubble flow:

$$|\vec{F}_d| = 2\pi d\eta V = \frac{1}{6}\pi d^3 \rho_f g = |\vec{F}_b|$$

Resulting velocity:

$$V = \frac{d^2 \rho_f g}{12\eta}$$

Example: 1 mm bubble in Ti (density 4100, viscosity 0.005):
V=0.67 m/s

Solid sphere Stokes flow:

$$|\vec{F}_d| = 3\pi d\eta V = \frac{1}{6}\pi d^3 (\rho_p - \rho_f) g = |\vec{F}_b|$$

Resulting velocity:

$$V = \frac{d^2 (\rho_p - \rho_f) g}{18\eta}$$

Example: 1 mm TiN (density 5200) in Ti: V=0.12 m/s

Diffusive Phenomena

	Diffusion	Heat conduction	Fluid flow
What's conserved?	Moles of each species	Joules of energy	kg m/s momentum
Local density	C	$\rho c_p T$	$\rho \vec{u}$
Units of flux	$\frac{\text{mol}}{\text{m}^2 \cdot \text{s}}$	$\frac{\text{W}}{\text{m}^2}$	$\frac{\text{kg}\frac{\text{m}}{\text{s}}}{\text{m}^2 \cdot \text{s}} = \frac{\text{N}}{\text{m}^2}$
Conservation equation*	$\frac{\partial C}{\partial t} = -\nabla \cdot \vec{J} + G$	$\rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot \vec{q} + \dot{q}$	$\frac{\partial(\rho \vec{u})}{\partial t} = -\nabla P - \nabla \cdot \tau + \vec{F}$
Constitutive equation	$\vec{J} = -D\nabla C$	$\vec{q} = -k\nabla T$	$\tau = -\mu \left[\nabla \vec{u} + (\nabla \vec{u})^T\right]$
Diffusivity	D	$\alpha = k/\rho c_p$	$\nu = \mu/\rho$
Result**	$\frac{\partial C}{\partial t} = D\nabla^2 C + G$	$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{q}}{\rho c_p}$	$\frac{\partial \vec{u}}{\partial t} = -\frac{\nabla P}{\rho} + \nu \nabla^2 \vec{u} + \frac{\vec{F}}{\rho}$
Convective flux	$C\vec{u}$	$\rho c_p T \vec{u}$	$\rho \vec{u} \vec{u}$
New result**	$\frac{DC}{Dt} = D\nabla^2 C + G$	$\frac{DT}{Dt} = \alpha \nabla^2 T + \frac{\dot{q}}{\rho c_p}$	$\frac{D\vec{u}}{Dt} = -\frac{\nabla P}{\rho} + \mu \nabla^2 \vec{u} + \frac{\vec{F}}{\rho}$

*Only considering diffusive fluxes. ^{T} in fluid constit. means matrix transpose. **For uniform properties.

Reynolds number as a ratio of kinetic energy/viscous drag
Reynolds number as a ratio of convective/viscous stresses

Extremely low Reynolds number (<0.1) -> eliminate convection

- Stokes Flow

Reynolds Number and Stability

Flow instability leading to turbulence

- Channel/couette flow: Re<1000 -> always laminar
- Tube flow: Re<2100 -> always laminar
- Falling film: high surface tension -> like channel/couette;
  low surface tension: Re<20 -> always laminar

Instability: if there are no perturbations, there's no turbulence...

Engineering and Society

Lessons from 9/11