3.044 Problem Set 2

Advanced Heat Conduction

Due Tuesday February 22, 2005

1. Finite difference model of 1-D unsteady conduction (34)

   The design for some optical device calls for a poly(methyl methacrylate) (PMMA, a.k.a. plexiglass) plate 5 mm thick at 30°C to be immersed in a hot liquid at 80°C. Because the index of refraction will change with temperature, the designer wants to know how the temperature varies across the plate as a function of time.

   Let x represent the distance into the PMMA plate from one side.

   Data:

   - PMMA thermal conductivity: $0.21\frac{\mathrm{W}}{\mathrm{m \cdot K}}$
   - PMMA density: $1190\frac{\mathrm{kg}}{\mathrm{m}^3}$
   - PMMA specific heat: $1470\frac{\mathrm{J}}{\mathrm{kg \cdot K}}$
   - Fluid heat transfer coefficient: $9000\frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$

   Data source: `http://www.asahi-kasei.co.jp/plastic/e/technical/pmma/bussei_doukou.htm`

   (a) Calculate the Biot number for this situation. What kind of boundary condition approximation can you make at the two sides of the plate? (4)

   (b) For the 1-D explicit form of the finite difference method, give the temperature of an interior node in terms of the mesh Fourier number and the temperatures of the adjacent nodes at the previous timestep. (5)

   (c) For a mesh with eight evenly-spaced nodes in the x-direction (7 intervals), what is the maximum allowed time step size Δt in the explicit scheme? How does this change for thirteen nodes (12 intervals)? (5)

   (d) Using a spreadsheet or a program in the language of your choice,[1] use the difference equations from part 1b to calculate the temperature profiles at times from zero to the steady-state timescale with seven intervals. Make a graph of your results and include it with the material you turn in. Also, submit the spreadsheet or code on the MIT server (10)

   (e) Repeat the finite difference calculation in part 1d for thirteen nodes (12 intervals, on the second sheet if a spreadsheet), again making a graph and submitting the spreadsheet or code. What do you notice about the temperature profiles with an even number of intervals at Fo_M=0.5? (10)

   For fun: increase the mesh Fourier number in your simulation to exceed the "maximum allowed" value and watch what happens to the temperatures.

[1] Note: your code or spreadsheet formulas will need to be readable and understandable by the grader; Perl geek code or similarly obfuscated material will not get credit.

2. Radiative cooling of an aluminum cube (20)

   An anodized aluminum object at 1000 K, which we'll model as a cube with size 0.1 m, is placed in a cold (much lower temperature), black enclosure. Its bottom is resting on an insulated surface, so it cools by radiation from its other five sides.

   Aluminum data:

   - Thermal conductivity: $k = 238 \frac{\text{W}}{\text{m}\cdot\text{K}}$
   - Density: $\rho = 2700 \frac{\text{kg}}{\text{m}^3}$
   - Heat capacity: $c_p = 917 \frac{\text{J}}{\text{kg}\cdot\text{K}}$
   - Anodized aluminum emissivity: $\epsilon = 0.85$

   (a) Express the net radiative heat flux from the cube surface as a function of cube surface temperature. You may assume the surroundings are at a low enough temperature that their emission back to the cube is negligible. (4)

   (b) Write an expression for the radiative "heat transfer coefficient", which is the heat flux divided by the surface temperature, using the grey body approximation, and an "environment" temperature of zero. (4)

   (c) Calculate the maximum Biot number for radiative cooling of this cube at between 1000 K and 400 K. What approximation may we use for the temperature profile across the cube? (4)

   (d) Calculate the time required for this object to cool from 1000 K to 400K by radiation alone. (8)

3. Dimensional analysis: resistance welding (24)

   We would like to simplify and generalize the temperature vs. location and time solution for resistance welding which we learned about in Problem Set 1 #4 in order to apply it easily to other materials and related processes. For medium to long timescales, the Gaussian is the relevant solution to the time-dependent heat equation:

$$T = T_{init} + \frac{\beta}{\rho c_p \sqrt{\pi \alpha t}} \exp\left(-\frac{x^2}{4\alpha t}\right)$$

   where β is half of the amount of heat per unit area in $\frac{\text{J}}{\text{m}^2}$ (equivalent to the product $\delta(T_{max} - T_{init})$ for a "hat function" initial condition of width 2δ).

   (a) Choose the relevant dimensions (*e.g.* T, x, t, α etc.), and write all of their base units. (4)

   (b) Write the number of dimensions and the number of base units, and use the Buckingham pi theorem to determine the number of dimensionless parameters. (3)

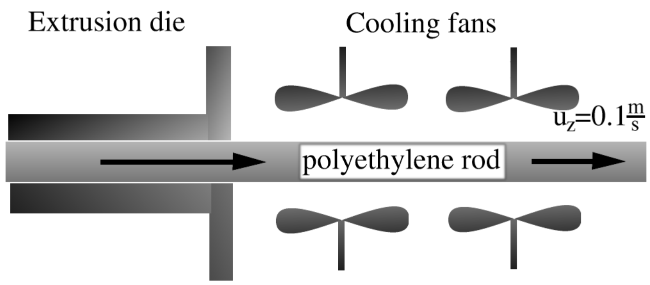
   (c) Construct your dimensionless parameters, keeping at least the temperature T and distance from the center x, and eliminating as many others as possible. (7)

   Note: you *may* have square roots, *i.e.* exponents which are multiples of $\frac{1}{2}$, not just integers.

   (d) Rewrite the Gaussian solution above in terms of your new dimensionless parameters, *i.e.* $\pi_T = ...$ Include all coefficients in all parts of the Gaussian solution. (5)

   (e) Draw a graph of this dimensionless solution, labeling the maximum dimensionless temperature and the width (e.g. full width at half maximum). (5)

4. Polymer extrusion and thermal stress (22)



A cylindrical high-density polyethylene rod 0.02 m in diameter is exiting an extruder at a uniform rate of $u_z = 0.1 \frac{\text{m}}{\text{sec}}$ and a temperature of 160° C. It is cooled by fans to room temperature which is 40° C, with a heat transfer coefficient of 130 $\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$.

Thermal stress is roughly proportional to temperature difference between the surface and center of the rod, so we want to estimate that temperature difference.

You may neglect thermal gradients in the lengthwise direction, and assume steady-state, so if we convert distance from the extruder to time $t = z/u_z$, the energy transport equation will be

$$r u_z \frac{\partial T}{\partial z} = \alpha \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right)$$

HDPE data: $k = 0.64 \frac{\text{W}}{\text{m} \cdot \text{K}}$, $\rho = 920 \frac{\text{kg}}{\text{m}^3}$, $c_p = 2300 \frac{\text{J}}{\text{kg} \cdot \text{K}}$.

(a) Calculate the Biot number for this situation using the radius as the lengthscale. (4)

(b) Calculate the Fourier number at $z = 0.33$m, $z = 1$m and $z = 3.3$m (hint: convert z to time). (5)

(c) Use the graphs provided in class to estimate the temperatures at the center of the rod and the surface at those three distances. Of these three, which gives the largest temperature difference between the surface and the center? (8)

(d) If your 3.032 thermal stress calculation tells you the temperature is too non-uniform and will lead to product defects, how can you correct this? That is, what simple design modification would make the temperature more uniform? (Hint: consider the shape of the T vs. r curves for various dimensionless numbers.) (5)