# 3.044 Test 1—Corrected

## Heat Transfer, Engineering Economics

Wednesday–Friday March 9–11, 2005

Welcome to the first 3.044 test of Spring 2005. Do not open this until you are told to begin.

- This is a 50-minute, in-class, closed-notes, closed book test, and you may not consult others during it, though you may use a calculator.
- You are expected to know and be able to use the equations on the review sheet. Other equations are given on the last page of this exam. If you need an equation which is neither on the review sheet nor on the equations page, ask and it will be written on the board.
- Feel free to use this test booklet as scratch paper, you can take it with you after the test, and you may ask for extra scratch paper or answer booklets at any time.
- Answer all of the questions, and be sure to show *all* work in your answer booklets, so if your numerical answer is incorrect, you might get partial credit for correct methodology and equations.
- Please begin your answers for each question on a new sheet of paper.
- You may answer the questions in any order.
- The test will be graded and scored, then returned to you along with a fresh test so you can correct it during the Thursday or Friday recitation. You may take as long as you like to correct the test, and if logistical difficulties prevent you from completing it in recitation, we will make arrangements for you to complete it at another time.
- Please indicate clearly in the Section space on the cover of your answer booklet when you will make your corrections, e.g. "Thursday recitation" or "Friday recitation".
- You may use any resources you like to help you to understand the material between the in-lecture portion and correction during recitation, including the instructor and TA (though we might not tell you exactly how to answer a question).
- Following the corrections, your test will be re-graded and scored. Your final score for the test will be the mean of these two scores.
- If you have questions, raise your hand where you are, and someone will come to answer them.

Knock it dead!

1. Write your name on all of your answer booklets (5 pts)

2. Heat transfer and injection molding (20 pts)

   The materials supplier PolyCorp is developing a chopped fiber polyethylene composite which can be injection molded directly into a water-cooled mold. They need you to help them estimate the time required for a part in a mold to cool to an acceptable temperature for removal.

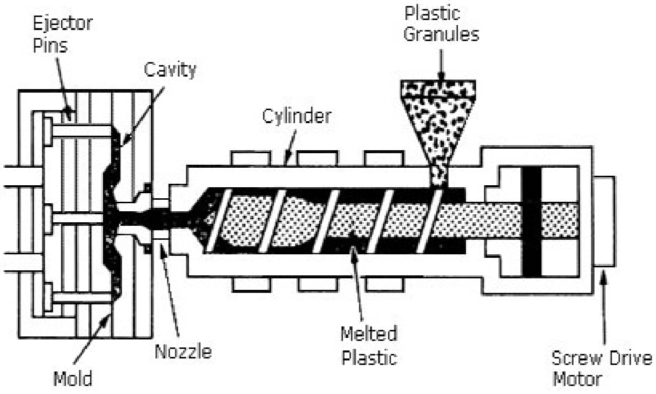


Diagram of a typical injection molding process.

(Image taken from the OSHA Technical Manual,

`http://www.osha-slc.gov/dts/osta/otm/otm_toc.html`.)

Chopped fiber composite properties and process parameters:

- Thermal conductivity: $k = 2.2 \frac{\mathrm{W}}{\mathrm{m \cdot K}}$
- Density: $\rho = 1100 \frac{\mathrm{kg}}{\mathrm{m}^3}$
- Average heat capacity*: $c_p = 3100 \frac{\mathrm{J}}{\mathrm{kg \cdot K}}$
- Injection temperature: $T_i = 225°\mathrm{C}$
- Cooling water temperature: $T_{fl} = 25°\mathrm{C}$
- Overall heat transfer coefficient (part to cooling water): $h = 880 \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}}$

*Averaged over the temperature range to approximately account for phase transformations.

(a) Calculate the Biot numbers for composite parts of 1 cm and 2 cm total thicknesses. What methodology can you use to estimate the centerline temperature? (6)

(b) Assuming the part is injected rapidly with uniform temperature, approximately how much time is required to reduce the centerline temperature of parts of 1 cm and 2 cm total thickness to 45°C? (8)

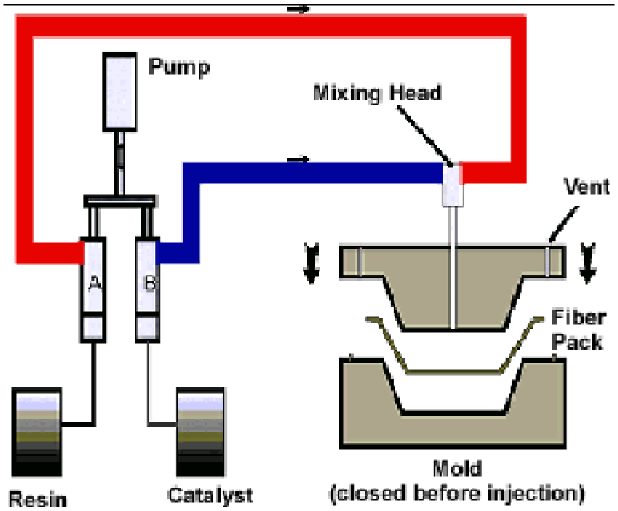
(c) Given these two data points, how does the cooling time scale with thickness in this Biot number range (*e.g.* linear, square root of thickness, etc.)? (6)

3. Cost, scale, and process selection (30 pts)

Company AeroForm manufactures high-performance carbon fiber composite parts for the commercial aircraft industry. Currently, these parts are manufactured using a resin-transfer molding process shown in the figure below. RTM involves six major process steps:

(a) Insert fiber preform (labeled "Fiber Pack" in the diagram) into the mold
(b) Close mold
(c) Inject mixed resin/catalyst
(d) Part solidifies via reaction
(e) Open mold
(f) Remove part

The cycle time for step (d) is approximately the same 25 minutes for parts of all sizes made by AeroForm. All other molding operations (a), (b), (c), (e) and (f) take a total of 5 minutes. Other relevant information is shown in the tables on the next page.



(a) How many RTM units are required to meet AeroForm's current demand of 5,000 units per year? (5)
(b) What is the unit cost ($/part) for AeroForm's current operations? Assume that AeroForm produces only this part and that they only incur materials and equipment costs. (10)

PolyCorp (from problem 2) has proposed a new processing option: injecting a chopped fiber composite into the mold. In principle, the new mateial does not change any process steps, except that there is no preform, and during step (d), instead of solidifying via reaction, the part hardens during cooling. The benefit of this new option would be greatly reduced cycle time. In fact, the cycle time for step (d) would drop from 25 minutes to the number you computed in problem 2b. (If you didn't get an answer to problem 2b, assume this takes 90 seconds.) Unfortunately, equipment cost is higher for this option at $1.8 million per machine, but total material costs are lower at $2/kg.

(c) At what production volume would it be cheaper for AeroForm to use PolyCorp's new material and process for parts of 1 cm total thickness? (15)

**Relevant Production Information**

| | | |
|---|---|---|
| Current Production Volume | 5,000 | units/year |
| Part thickness | 1 | cm |
| Part weight | 2 | kg |
| **Plant Operating Time** | | |
| Days per year | 250 | |
| Hours per day | 10 | |
| **Financial Parameters** | | |
| Discount rate | 15% | |
| Equipment life | 5 | years |

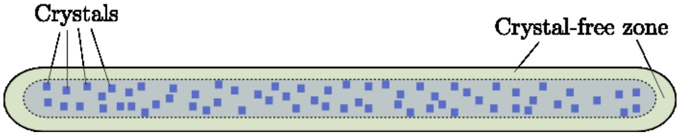
**Process Characteristics**

| | | |
|---|---|---|
| **RTM** | | |
| Materials Cost | 4 | $/kg |
| Process yield | 85% | good parts/parts processed |
| Equipment Cost | $1,000,000 | |
| Cycle times: | | |
| Reaction time (step d) | 1500 | seconds |
| Total of other cycle times | 300 | seconds |
| **Chopped Fiber Injection Molding** | | |
| Materials Cost | 2 | $/kg |
| Process yield | 85% | good parts/parts processed |
| Equipment Cost | $1,800,000 | |
| Cycle times: | | |
| Cooling time | Prob. 2b (1cm) or 90 | seconds |
| Total of other cycle times | 300 | seconds |

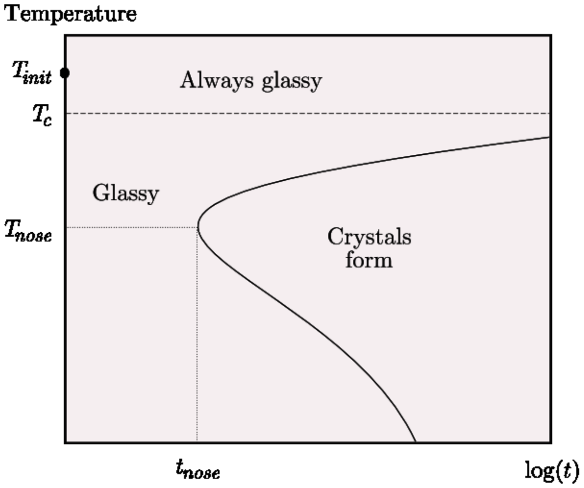
4. Crystal-free zone in a glass-ceramic dish (22 pts)

**Background** Some dishes, such as the Corelle line by Corning, are made of a high-tech glass-ceramic material in which partial crystallization of the interior puts the surface in compression, resulting in high bending strength for a thin heat-resistant dish. The glassy phase is stable above a temperature $T_c$, and the crystalline phase below that temperature.

During cooling by immersion in a fluid at temperature $T_{fl}$, a surface layer cools quickly enough to avoid any crystallization. The criterion for this is roughly that the temperature must fall below $T_{nose}$ before time reaches $t_{nose}$, as indicated in the T-T-T (time-temperature-transformation) diagram below.



Glass-ceramic "dish" (not to scale)



T-T-T (time-temperature-transformation) diagram

Properties of the glassy phase and process parameters:

- Thermal conductivity: $k = 0.4 \frac{\text{W}}{\text{m}\cdot\text{K}}$
- Density: $\rho = 2400 \frac{\text{kg}}{\text{m}^3}$
- Heat capacity: $c_p = 900 \frac{\text{J}}{\text{kg}\cdot\text{K}}$
- Heat transfer coefficient: $h = 3000 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$
- Dish thickness: $L = 0.01\text{m}$ (a thick dish)
- Temperatures: $T_{init} = 1000\text{K}$, $T_c = 900\text{K}$, $T_{nose} = 720\text{K}$, $T_{fl} = 300\text{K}$; time $t_{nose} = 4$ seconds

(a) Sketch the temperature distribution during cooling, ignoring phase transformations. (6)

(b) Give an expression for the temperature as a function of distance into the plate which is approximately valid at short time scales. (4)

(c) Using your expression from part 4b, estimate the distance into the plate where temperature is $T_{nose}$ when time is $t_{nose}$. (8)

(d) Determine whether your expression in part 4b (and thus your calculation in part 4c) is valid at time $t_{nose}$. (4)

5. Dimensional analysis: mixed radiation and conduction (23 pts)

   A number of heat transfer situations involve quasi-steady 1-D conduction through a solid shell which radiates from its outer surface into a cold dark environment. For these situations, the inner surface temperature is so much higher than the environment temperature that one can assume negligible heat return from the surroundings, either by reflection or radiation, so the "environment temperature" is effectively zero.

   Examples might include a high-temperature tool steel casting, in which solidification time is limited by conduction through a ceramic mold and radiation to the surroundings; or cooling of the young hot planet Earth limited by conduction through a thin solid crust and radiation into space (this will work as long as the Earth is young and hot enough that solar heating is a lot smaller than radiative cooling).

   Because the analytical solution is complicated, the goal here is to come up with a simplified curve, or set of curves, which allow one to calculate heat flux from other parameters in this general class of situations.

   (a) Write the heat flux in terms of other problem parameters in the form $A = f(B, C, ...)$, including shell thickness and its property(ies), and group radiative emissivity together with the radiation constant as a single parameter $\epsilon\sigma$. Note that there is only one known temperature, which is on the hot side of the shell. (4)

   (b) Count the number of parameters and base units, and use the Buckingham pi theorem to determine the number of independent dimensionless parameters. (4)

   (c) Choose which parameters to eliminate, and form your dimensionless flux and the other dimensionless parameter(s). Try to choose such that the resulting dimensionless parameters are not too complicated. (10)

   (d) How do you expect the dimensionless flux to behave when the other dimensionless parameter(s) is/are very small, and very large? (5)

# Equation sheet

- Error function (complement) solution to the time-dependent 1-D heat equation:

$$\frac{T-T_s}{T_i-T_s}=\operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right),\quad \frac{T-T_i}{T_s-T_i}=\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

Error function table:

| $x$ | erf($x$) | erfc($x$) | $x$ | erf($x$) | erfc($x$) | $x$ | erf($x$) | erfc($x$) |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0.65 | 0.642029 | 0.357971 | 1.6 | 0.976348 | 0.023652 |
| 0.05 | 0.056372 | 0.943628 | 0.7 | 0.677801 | 0.322199 | 1.7 | 0.983790 | 0.016210 |
| 0.1 | 0.112463 | 0.887537 | 0.75 | 0.711156 | 0.288844 | 1.8 | 0.989091 | 0.010909 |
| 0.15 | 0.167996 | 0.832004 | 0.8 | 0.742101 | 0.257899 | 1.9 | 0.992790 | 0.007210 |
| 0.2 | 0.222703 | 0.777297 | 0.85 | 0.770668 | 0.229332 | 2.0 | 0.995322 | 0.004678 |
| 0.25 | 0.276326 | 0.723674 | 0.9 | 0.796908 | 0.203092 | 2.1 | 0.997021 | 0.002979 |
| 0.3 | 0.328627 | 0.671373 | 0.95 | 0.820891 | 0.179109 | 2.2 | 0.998137 | 0.001863 |
| 0.35 | 0.379382 | 0.620618 | 1.0 | 0.842701 | 0.157299 | 2.3 | 0.998857 | 0.001143 |
| 0.4 | 0.428392 | 0.571608 | 1.1 | 0.880205 | 0.119795 | 2.4 | 0.999311 | 0.000689 |
| 0.45 | 0.475482 | 0.524518 | 1.2 | 0.910314 | 0.089686 | 2.5 | 0.999593 | 0.000407 |
| 0.5 | 0.520500 | 0.479500 | 1.3 | 0.934008 | 0.065992 | 2.6 | 0.999764 | 0.000236 |
| 0.55 | 0.563323 | 0.436677 | 1.4 | 0.952285 | 0.047715 | 2.8 | 0.999925 | 0.000075 |
| 0.6 | 0.603856 | 0.396144 | 1.5 | 0.966105 | 0.033895 | 3.0 | 0.999978 | 0.000022 |

- Gaussian solution to the time-dependent 1-D heat equation:

$$T=T_i+\frac{(T_0-T_i)\delta}{\sqrt{\pi\alpha t}}\exp\left(-\frac{x^2}{4\alpha t}\right)$$

- Semi-infinite criterion:

$$\frac{L}{2\sqrt{\alpha t}}\geq 2$$

- Newtonian cooling:

$$\frac{T-T_{fl}}{T_i-T_{fl}}=\exp\left(-\frac{Aht}{V\rho c_p}\right)$$

- Unit cost:

$$\text{Unit cost}=\frac{\text{Total annual equivalent cost}}{\text{Annual production volume}}$$

$$\text{Total annual equivalent cost}=\text{Annual equipment cost}+\text{Annual material cost}$$

- Cooling curves for the center of a plate ($L$ is half the thickness if cooled from both sides):