Friction Factors, Boundary Layers

3.044 March 28, 2005

Mechanics:

- Problem Set 4 due Wednesday
- Problem Set 5 due next Monday

Today's lecture:

- Friction factor and drag in a tube
- Boundary layer, friction factor and drag on a flat plate

Inclusions and Bubbles: Drag Force

Roughly speaking: sum of viscous drag and kinetic energy terms

Sphere:

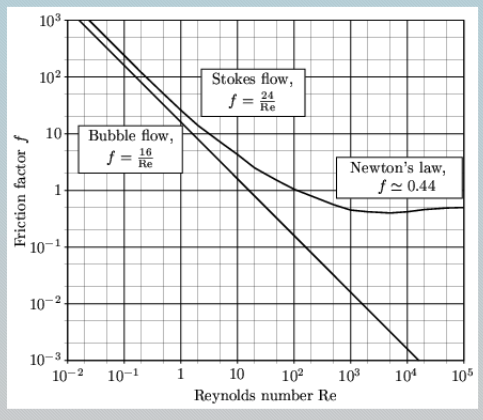
$$F_d = 3\pi d\eta V + 0.44 \cdot \frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2$$

Bubble:

$$F_d = 2\pi d\eta V$$

Ratio of kinetic energy to viscous contribution: Reynolds number

$$\mathrm{Re} = \frac{\rho V d}{\eta}$$



General friction factor:

$$F_d = fKA = f \cdot \frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2$$

Solid extremes: $f = \frac{24}{\mathrm{Re}} \Rightarrow f = c_d$

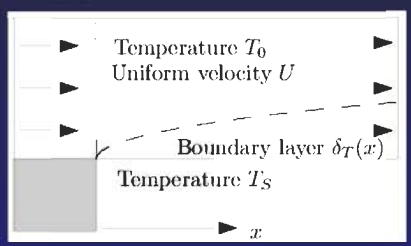
Tube Flow Friction Factor

Image removed for copyright reasons.

Please see:
Fig. 3.2 in Poirier, D. R., and G. H. Geiger. *Transport Phenomena in Materials Processing*. Warrendale, PA: The Minerals, Metals and Materials Society, 1994.

Boundary Layers

Conduction in moving solid



Steady-state, $\delta_T << x$

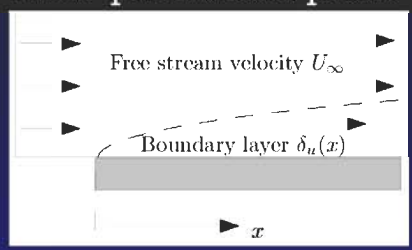
$$u_x \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$T = A + B\text{erf}\left(\frac{y}{2\sqrt{\alpha x/U_\infty}}\right)$$

1% BL profile:

$$\delta_T = 3.6\sqrt{\alpha x/U_\infty}$$

Flow past a flat plate



1% BL profile:

$$\delta_u = 5.0\sqrt{\nu x/U_\infty}$$

Why 5.0, not 3.6?
Continuity: y-veloc > 0, carries slow fluid upward
Convective momentum transport!