Chapter 2

Heat Conduction

2.1 February 4, 2005: Start heat conduction

Mechanics:

- Test conflicts? March 9–11, April 20–22. If no, will get a room.
- Handouts: ABET, PS1 due Mon 2/14.
- Interesting lecture: ASM dinner Chiang "Ceramics in Electrochemical Systems" Thu Feb 17; student $8, RSVP Sam Davis.

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Open: Colleen's facility with names, my advisee...

Heat Conduction Analogy to diffusion: Conservation of thermal energy.

$$\text{accumulation} = \text{in} - \text{out} + \text{generation}$$

$$V\frac{dH}{dt} = Aq_{\text{in}} - Aq_{\text{out}} + V\dot{q}$$

Note on the accumulation term: when temperature changes, enthalpy changes according to the heat capacity, build up units from dT/dt (Kelvin/sec) adding c_p and ρ to get to Joules/sec.

What's heat flux $\vec{q}$? Like diffusion goes down the conc gradient (actually, chem potential gradient), heat goes down the temperature gradient, proportionality constant k:

$$\vec{q} = -k\nabla T. \tag{2.1}$$

Using that in-out and that accumulation term, derive the 1-D heat equation, same as diffusion. Simplify constant k, 1-D, so:

$$\rho c_p\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \dot{q}. \tag{2.2}$$

Define thermal diffusivity $\alpha = k/\rho c_p$, with no gen reduces to diffusion equation, and give 1-D solutions:

- 1-D steady-state: linear temperature. Result: flux $q_x = k\Delta T/L$

- Cylindrical steady-state: different control volumes, $T = A \ln r + B$.

  Total flux: $q_r = -k\partial T/\partial r = A/r$, $2\pi r L q_r = 2\pi A L$.

  So, C_{out} at outside, C_{in} at inside, what to do between? Use R_1 and R_2 for inner, outer radii. Fick's first, assume 1-D, so C is function of r only.

  $$J_r = -D\frac{dC}{dr} \tag{2.3}$$

  Conservation: in at $r + \Delta r$, out at r, no gen or accum, area $2\pi r L$:

  $$0 = [2\pi r L J_r]_r - [2\pi r L J_r]_{r+\Delta r}, \tag{2.4}$$

  divide by $2\pi L$, $\Delta r \to 0$:

  $$0 = -\frac{d}{dr}[rJ_r] \tag{2.5}$$

  Plug in flux:

  $$0 = \frac{d}{dr}\left(rD\frac{dC}{dr}\right). \tag{2.6}$$

  Now solve:

  $$A' = rD\frac{dC}{dr} \tag{2.7}$$
  $$\frac{A'}{Dr} = \frac{dC}{dr} \tag{2.8}$$
  $$C = A\ln r + B \tag{2.9}$$

  where $A = A'/D$. From BCs:

  $$\frac{C - C_{in}}{C_{out} - C_{in}} = \frac{\ln(r/R_1)}{\ln(R_2/R_1)} \tag{2.10}$$

  Check at R_1 and R_2, units.

  Flux$= -DdC/dr$:

  $$J_r = -D\frac{dC}{dr} = -D\frac{d}{dr}\left[C_i n + (C_{out} - C_{in})\frac{\ln(r/R_1)}{\ln(R_2/R_1)}\right] = D\frac{C_{in} - C_{out}}{\ln(R_2/R_1)}\frac{1}{r}. \tag{2.11}$$

  Important result: not flux, but flux times area.

  $$AJ_r = -2\pi r L D\frac{dC}{dr} = 2\pi r L D\frac{C_{in} - C_{out}}{\ln(R_2/R_1)}\frac{1}{r}. \tag{2.12}$$

  Note rs cancel, so AJ_r is constant for all r. Make sure units work. Cool.

- Spherical steady-state: still another, $T = \frac{A}{r} + B - \dot{q}r^2/6k$.