

# 3.044 Test 2

## Liquid-Solid Processing, Fluid Behavior

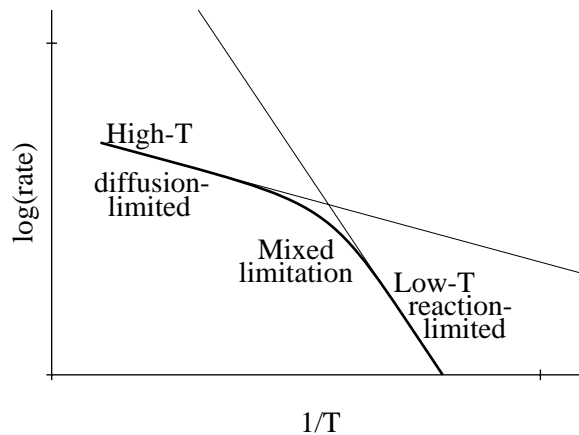
### Solutions

1. Write your name on all of your answer booklets

Someone wrote his name not just on every answer booklet, but on every page! Should I dock him two points?

2. Electroless nickel deposition

- (a) The deposition is limited by diffusion in the solution ahead of the advancing solidification front, which is an unstable situation. Put differently a bump on the nickel surface will have higher nickel ion flux (thinner boundary layer, surrounded by nickel-rich fluid, closer concentration isoquants, there are a number of ways to think about this) and grow faster than the region around it.
- (b) Diffusion-limited means the reaction is fast and diffusion is slow  $k'' \gg h_D$ . To go from diffusion-limited to reaction-limited operation, we want to enhance mass transfer relative to the reaction rate such that  $h_D \gg k''$ . There are several ways to do this, including:
  - Because activation energy of chemical reactions is generally a lot larger than that of diffusion, reactions are more strongly temperature-dependent. Lowering the temperature will thus lower  $k''$  more than it lowers  $h_D$ , which could make deposition reaction-limited (if the solution doesn't freeze first). This can be illustrated using an Arrhenius plot:



Reaction-Diffusion Arrhenius Plot

- To accelerate mass transfer, one can stir the solution or move the substrate, increasing the mass transfer coefficient  $h_D$ . Put differently, this decreases the boundary layer thickness, decreasing the diffusion lengthscale in the Biot number, driving the system toward reaction limitation.

- Slow down the reaction rate, either by reducing the concentration of reducing agent, using a slower/weaker reducing agent than sodium hypophosphite, or using an inhibitive additive of some kind, a sort of reverse catalyst. (Note “catalytic reduction” in the description describes the catalytic activity of the solid nickel surface, such that the reaction happens there and does not precipitate nickel powder throughout the solution.)

Any of these received full credit.

When reaction-limited, nickel ions will have the same concentration everywhere in solution right up to the substrate surface, the nickel film will grow at a uniform rate and the interface will be stable.

### 3. Tube flow calculation

To calculate the pressure drop for fully-developed flow in a tube:

- Calculate the Reynolds number using the diameter as the lengthscale and average velocity  $u_{av} = Q/A_{xs}$ ,  $A_{xs} = \pi R^2$ :

$$\text{Re} = \frac{\rho u_{av} d}{\mu}.$$

- If Reynolds number is less than 2100, flow is laminar, so just solve the Hagen-Poiseuille equation for  $\Delta P$ :

$$Q = \frac{\pi \Delta P R^4}{8\mu L} \Rightarrow \Delta P = \frac{8\mu L Q}{\pi R^4}.$$

This is easier than using the friction factor, so skip to the fully-developed part if done this way.

- Whether laminar or turbulent, one can use a graph to calculate the friction factor  $f$  from the Reynolds number and roughness/diameter ratio  $\epsilon/d$ ; if turbulent, roughness will increase friction factor.
- Calculate the drag force from the friction factor:

$$F_d = fKA = f \cdot \frac{1}{2} \rho u_{av}^2 \cdot 2\pi RL.$$

- Pressure drop is equal to drag force divided by cross-section area:  $\Delta P = \frac{F_d}{\pi R^2}$ .
- This is only valid for fully-developed flow so the tube must be much longer than the entrance length  $L \gg L_e$ . Entrance length can be calculated from the Reynolds number:

$$\frac{L_e}{d} = f(\text{Re}) = f\left(\frac{\rho u_{av} d}{\mu}\right).$$

These were worth two points each; omission of Hagen-Poiseuille was accepted if the answer mentioned use of the friction factor in the laminar case.

### 4. Diffusion of water vapor into a flowing adhesive sheet

- (a) Time required to flow down the exposed length is that length divided by velocity on top of the film, which is the maximum velocity:  $t = L_{exp}/u_{max}$ . Taking  $x$  as the flow direction and  $y$  as the distance from the top surface, maximum velocity is given by:

$$u_{max} = u_x|_{y=0} = \frac{\rho g \sin \theta L^2}{2\mu} = \frac{1060 \frac{\text{kg}}{\text{m}^3} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 0.707 \cdot (0.005\text{m})^2}{2 \cdot 1 \frac{\text{kg}}{\text{m}\cdot\text{s}}} = 0.0918 \frac{\text{m}}{\text{s}}.$$

$$t = \frac{L_{exp}}{u_{max}} = \frac{0.05\text{m}}{0.0918 \frac{\text{m}}{\text{s}}} = 0.545 \text{seconds}.$$

Is flow laminar? Check the Reynolds number using  $u_{av} = \frac{2}{3}u_{max}$  as the velocity:

$$Re = \frac{\rho u_{av} L}{\mu} = \frac{1060 \frac{\text{kg}}{\text{m}^3} \cdot 0.0612 \frac{\text{m}}{\text{s}} \cdot 0.005 \text{m}}{1 \frac{\text{kg}}{\text{m} \cdot \text{s}}} = 0.324.$$

Since this is below 20, we're well within the laminar flow régime.

- (b) For an order of magnitude, one can just estimate the diffusion length using:

$$L_D \simeq \sqrt{Dt} = \sqrt{10^{-9} \frac{\text{m}^2}{\text{s}} \cdot 0.545 \text{s}} = 2.3 \times 10^{-5} \text{m}.$$

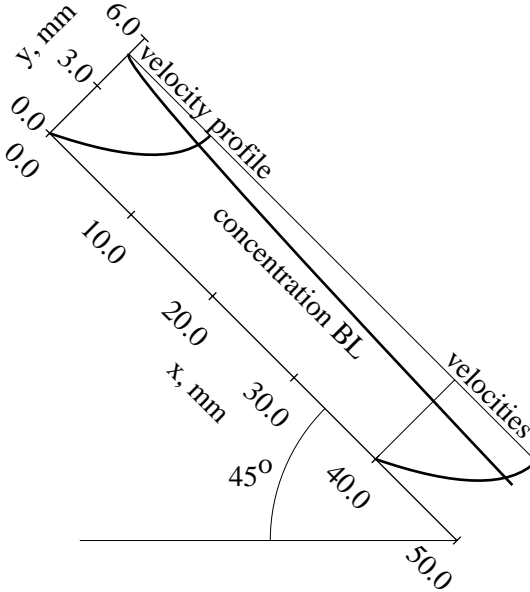
About 20 microns, which is quite shallow. Even if a small amount of water diffuses several times more deeply, this is still well within the top region with roughly uniform velocity.

- (c) To determine the rate-limiting step, check the Biot number, using the diffusion depth as the lengthscale:

$$Bi = \frac{k''L}{D} = \frac{5 \frac{\text{m}}{\text{s}} \cdot 2.3 \times 10^{-5} \text{m}}{10^{-9} \frac{\text{m}^2}{\text{s}}} \simeq 10^5.$$

There's no question this is diffusion-limited.

- (d) Your sketches should have looked something like:



- (e) Since this is clearly diffusion-limited, the top surface will rapidly reach equilibrium with the air, giving a constant top surface concentration of water. The initial condition has a uniform (and hopefully low) concentration of water.

Since the diffusion depth is tiny compared to the adhesive film thickness, and the top surface is a free surface where  $\partial u_x / \partial y = 0$ , the velocity is roughly uniform and equal to  $u_{max}$  throughout the diffusion boundary layer.

These and the much thinner boundary layer thickness than exposed surface length  $\delta_C \ll L_{exp}$ , give us the conditions for using the error function to describe diffusion in the  $y$ -direction from the top, with time replaced by exposure time  $t = x/u_{max}$ . Since the surface concentration is higher than the initial concentration, it is convenient to use the erfc form:

$$\frac{C - C_{init}}{C_S - C_{init}} = \text{erfc} \left( \frac{y}{2\sqrt{Dx/u_{max}}} \right).$$

## 5. Fluidized bed

- (a) Because drag force increases with velocity, a higher velocity on one side than the other will be opposed by a higher drag force, and this will balance the velocity across the bed. This is especially true for high flow rates where drag force increases faster than linearly.

A couple of students noted that higher drag force could push the particles up more strongly, destabilizing the bed. This is a problem if the bed is shallow relative to its width, but that should not be a problem with the dimensions given here.

- (b) The diagram included a “funnel” region in the bottom which is important to stability. If the particle bed shifts downward, more of it will be in the narrower and higher-velocity region, so the force will be higher and it will be lifted up. If it shifts upward, it will all be in the large-diameter region, with lower average velocity and lower force. So as long as it is partly in the funnel, the fluidized bed will be vertically stable.
- (c) This is a straightforward application of the tube bundle approximation equation for specific permeability:

$$S_0 = \frac{\text{solid area}}{\text{solid volume}} = \frac{\pi d^2}{\frac{1}{6}\pi d^3} = \frac{6}{d} = 3 \times 10^4 \text{m}^{-1}.$$

$$\mathcal{P} = K_1 \frac{\omega^3}{S_0^2(1-\omega)^2} = \frac{1}{4.2} \frac{0.45^3}{(3 \times 10^4 \text{m}^{-1})^2 \cdot (1-0.45)^2} = 8.0 \times 10^{-11} \text{m}^2.$$

- (d) Flow rate calculation is a matter of using Darcy’s Law, which you were asked to memorize:

$$V_0 = \frac{Q}{A} = \frac{\mathcal{P} \Delta P'}{\mu L}.$$

What to use for  $\Delta P'$ ? That will be the pressure difference plus the weight of the fluid (air). In a fluidized bed, by definition, the pressure difference is that required to support the weight of the particle bed, which will probably be much larger than the weight of the air (by about three orders of magnitude), so the fluid weight can be safely neglected.

Pressure is force per unit area, and the weight per unit area of the fluidized bed will be equal to solid density times solid fraction times gravitational acceleration times (volume divided by cross section area = length):

$$\Delta P' \simeq \Delta P = \rho_p g(1-\omega)L.$$

Substitute this back into Darcy’s law:

$$V_0 = \frac{Q}{A} = \frac{\mathcal{P} \rho_p g(1-\omega)L}{\mu L} = \frac{\mathcal{P} \rho_p g(1-\omega)}{\mu} = \frac{8.0 \times 10^{-11} \text{m}^2 \cdot 3600 \frac{\text{kg}}{\text{m}^3} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot (1-0.45)}{10^{-5} \frac{\text{kg}}{\text{m}\cdot\text{s}}} = 0.155 \frac{\text{m}}{\text{s}}.$$

This times the cross-section area  $A_{xs} = \pi(0.25\text{m})^2$  gives a flow rate of  $0.030 \text{m}^3/\text{second}$ .

Note that this superficial velocity and flow rate are independent of the height of the bed  $L$ , so it need not be adjusted as the ceramic is slowly depleted by forming the casting molds.

- (e) The Reynolds number for flow through a porous medium is given by:

$$\text{Re} = \frac{\rho_f V_0}{\mu(1-\omega)S_0} = \frac{1.9 \frac{\text{kg}}{\text{m}^3} \cdot 0.155 \frac{\text{m}}{\text{s}}}{10^{-5} \frac{\text{kg}}{\text{m}\cdot\text{s}} \cdot (1-0.45) \cdot 3 \times 10^4 \text{m}^{-1}} = 1.78.$$

This does not quite satisfy the validity criterion for the tube bundle approximation, so the above calculations are not quite valid. Pressure drop should be a bit higher for a given flow rate, so flow rate should be a bit lower. (In other words, the author of this problem messed up...)

Then again, given the shape of the  $f$ -Re curve for porous media, it’s not far off.