

# 3.044 Problem Set 1

## Heat Conduction

Due Monday February 14, 2005

1. Thermal properties and optimal materials selection (30 pts, 5 each)

In many situations, product designs include parts whose only function is to conduct or resist the conduction of heat. Materials for these parts are thus chosen entirely on the basis of their thermal properties. Select the best material for each of the following applications using the “MatML Grapher” software at <http://orbis.kent.edu/matdl/matml/select.php> supplemented with the extra candidate materials below. (Select the conductivity/thermal density graph to make things easier.)

- (a) Heat shield sandwiched between a hot body and a cold one which minimizes the steady flux between them.
- (b) Heat shield which protects something from short, intense bursts of heat (long timescale is needed).
- (c) Cheap (i.e. not diamond) temperature sensor, in which short timescale of heat conduction is necessary for rapid response.
- (d) Light heat reservoir which must hold as much heat as possible per degree C per unit weight.
- (e) Heat sink for a semiconductor device, which must minimize temperature difference for a given flux.
- (f) Heat sink for melt spinning, in which liquid metal is injected against a rotating heat sink where it is solidified as rapidly as possible, so the material must conduct heat away from the surface quickly. (Hint: evaluate the flux through  $x = 0$  in an erfc-like unsteady conduction problem. Diamond is not an economically viable option.)

Extra candidate materials:

Material	$k, \frac{\text{W}}{\text{m}\cdot\text{K}}$	$\rho, \frac{\text{g}}{\text{cm}^3}$	$c_p, \frac{\text{J}}{\text{kg}\cdot\text{K}}$
gold	315.5	19.3	130
silver	425	10.5	234
diamond	2320	3.5	519
graphite	63	2.25	711
lime (CaO)	15.5	3.32	749
alumina ( $\text{Al}_2\text{O}_3$ )	39	3.96	804

For those interested: Ashby’s appendix has a plot similar to that in the MatML Grapher, but with  $k$  vs.  $\alpha$  (which they call  $\lambda$  vs  $a$ ). Which format do you consider more visually helpful: that, or the Grapher’s  $k$  vs.  $\rho c_p$ ?

For more flexible materials selection, try: [http://orbis.kent.edu/matdl/matml/select\\_other.php](http://orbis.kent.edu/matdl/matml/select_other.php)

2. Layered furnace wall and British units (20 pts)

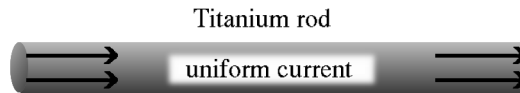
A cylindrical furnace wall consists of an inner graphite layer 18 inches (1.5 feet) thick, followed by an outer layer of insulating brick four feet thick, both surrounding a cylindrical container 15 feet high (on the inside) with an inner diameter of 20 feet containing molten metal.

Data:

- Outer brick-environment heat transfer coefficient: 4 BTU/hr·ft<sup>2</sup>·°F
  - Brick thermal conductivity: 16 BTU/hr·ft·°F
  - Graphite thermal conductivity:  $63 \frac{\text{W}}{\text{m}\cdot\text{K}}$  times  $0.557 \frac{\text{m}\cdot\text{K}}{\text{W}} \frac{\text{BTU}}{\text{hr}\cdot\text{ft}\cdot\text{°F}}$
  - Inner graphite-liquid metal heat transfer coefficient: very large (assume infinite)
- (a) Calculate the total heat loss in BTU/hr from molten metal at 2000°F to an environment at 70°F. (You may neglect the corners of the cylinder, but include the top and bottom.) (8)
- (b) How would your solution to part 2a would change if you include heat conduction in the corners? (*I.e.* is your answer an overestimate or underestimate?) (4)
- (c) Calculate the temperature at each of the interfaces in the radial direction. (8)

1 BTU (British Thermal Unit) of heat raises the temperature of 1 pound of water by 1°F.

3. Joule heating of a titanium rod (25 pts)



At time  $t = 0$ , the temperature in a titanium rod 0.25 cm ( $2.5 \times 10^{-3}$  m) in diameter is uniform and equal to its surface temperature. A switch is turned on so that a current passes lengthwise through the rod, heating it through a mechanism known as “Joule heating”. Assume the current is uniformly distributed throughout the rod, so it is heated uniformly, with a uniform heat generation rate per unit volume  $\dot{q}$ .

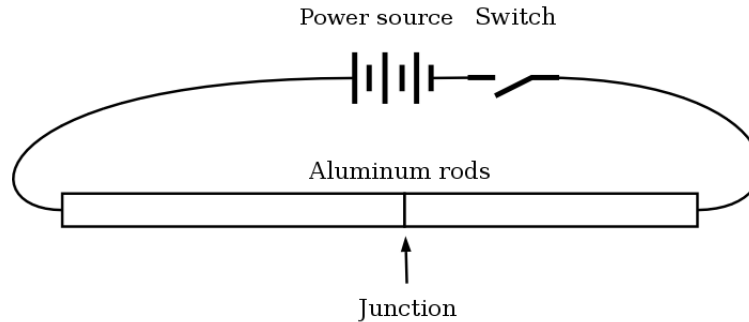
Titanium data:

- Thermal conductivity:  $k = 20 \frac{\text{W}}{\text{m}\cdot\text{K}}$
  - Density:  $\rho = 4700 \frac{\text{kg}}{\text{m}^3}$
  - Heat capacity:  $c_p = 700 \frac{\text{J}}{\text{kg}\cdot\text{K}}$
- (a) At a current of 10 amperes, the heat generation rate per unit volume  $\dot{q}$  is approximately  $5 \times 10^6 \frac{\text{W}}{\text{m}^3}$ . Use the uniform-generation solution to the heat conduction equation to calculate the maximum temperature difference between the center of the rod and its surface. (8)
- (b) If the current through the rod doubles to 20 amps, the heat generation rate  $\dot{q}$  quadruples. How does this affect the difference between the maximum temperature in the center and the temperature at the surface? (4)
- (c) Assuming the surface is held at a constant temperature, sketch the temperature  $T$  as a function of  $r$  across the rod for several different times  $t$ . (This should have one curve for  $t = 0$ , one for the steady-state at  $t = \infty$ , and others for several intermediate times.) (8)
- (d) Approximately how long will it take for the center of the rod to reach its maximum temperature? (5)

4. Heat Transfer in Resistance Welding (25 pts)

In resistance welding, two pieces of metal are held against each other, and a very high voltage is applied between them for a very short time. The highest resistance in the circuit is at the interface, so heat concentrates there, melting the metal and joining the two pieces together.

Two cylindrical aluminum rods are joined end-to-end in this way. You would like to know the temperature profile as a function of time  $T(x, t)$  to assess the width of the heat-affected zone (region with significant microstructure change due to heating).



Aluminum data:

- Thermal conductivity:  $k = 238 \frac{\text{W}}{\text{m}\cdot\text{K}}$
- Density:  $\rho = 2700 \frac{\text{kg}}{\text{m}^3} (= 2.7 \frac{\text{g}}{\text{cm}^3})$
- Heat capacity:  $c_p = 917 \frac{\text{J}}{\text{kg}\cdot\text{K}}$

- (a) Let  $t = 0$  at the moment immediately after the very brief application of current (*i.e.* at  $t = 0$  the current is turned off). Sketch the temperature distribution at several times starting at  $t = 0$ . (7)
- (b) Assuming there is no heat loss from the sides of the rod, write the heat conduction equation solution which describes the temperature distribution at moderate to long time scales (while the rod length can still be considered infinite). (4)
- (c) At  $t = 0$ , the whole rod is at  $40^\circ\text{C}$  due to Joule heating by the current. Another  $3 \times 10^6 \text{ J/m}^2$  of Joule heating energy has been deposited right at the junction to make the weld. Calculate the maximum temperature at  $t = 1$  seconds and  $t = 10$  seconds. (Hint in applying this solution: consider the relationship between energy or enthalpy and temperature.) (7)
- (d) Also at  $t = 1$  seconds, what is the width of the region where the temperature difference  $T - 40^\circ\text{C}$  is at least half of its maximum value? What is that width at  $t = 10$  seconds? (7)