March 16, 2005: Sphere drag and Reynolds number

Mechanics:

- PS4 due 3/30
- Course evals today...

Muddy from last time:

- Flow rate and average velocity. Last time looked at flow through tube, result was Hägen-Poisseuille: equation:

$$Q = \frac{\pi(P_1 - P_2)}{2\mu L}\left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{\pi(P_1 - P_2)R^4}{8\mu L}. \tag{4.32}$$

  Didn't really discuss units, this is in volume/time, m^3/seconds.

  Also, gives average velocity in confined flows:

$$u_{av} = \frac{Q}{A_{xs}} = \frac{\frac{\pi(P_1-P_2)R^4}{8\mu L}}{\pi R^2} = \frac{(P_1 - P_2)R^2}{8\mu L}. \tag{4.33}$$

  Recall that the velocity profile was:

$$u_z = \frac{P_1 - P_2}{4\mu L}(R^2 - r^2), \tag{4.34}$$

  with a maximum at $r = 0$. So for a tube, average velocity is *half* of maximum velocity.

  What about flow down a plane, or channel flow?

$$u_x = \frac{g\sin\theta}{2\nu}(2Ly - y^2) \tag{4.35}$$

  Flow rate: Q, volume per unit time through a surface. If width of the falling film is W, then flow rate is:

$$Q = \int_S \vec{u}\cdot\hat{n}dA = \int_{y=0}^{L} u_x W dy = \frac{Wg\sin\theta}{\nu}\left[\frac{Ly^2}{2} - \frac{y^3}{6}\right] = \frac{Wg\sin\theta}{\nu}\frac{L^3}{3} \tag{4.36}$$

  Average velocity is Q/A, in this case Q/LW:

$$u_{av} = \frac{Q}{LW} = \frac{g\sin\theta L^2}{3\nu}; \tag{4.37}$$

$$u_{max} = u_x|_{y=L} = \frac{g\sin\theta L^2}{2\nu}. \tag{4.38}$$

  So average velocity is 2/3 of maximum for falling film, channel flow, etc.

Reynolds number Dimensional analysis:

$$|\vec{F}_d| = f(V, \mu, d, \rho)$$

Five parameters, three base units, so two dimensionless parameters. Four different nondimensionalizations!

Keep	π_F	π_{other}
F, μ	$\frac{F}{\rho V^2 d^2}$	$\pi_\mu = \frac{\mu}{\rho V d}$
F, ρ	$\frac{F}{\mu V d}$	$\pi_\rho = \frac{\rho V d}{\mu}$
F, V	$\frac{F\rho}{\mu^2}$	$\pi_V = \frac{\rho V d}{\mu}$
F, d	$\frac{F\rho}{\mu^2}$	$\pi_d = \frac{\rho V d}{\mu}$

The first is the ratio of total force to the product of dynamic pressure and area, the kinetic energy drag term. The second is the ratio of total force to the viscous drag. The third and last are identical, and relatively useless physically. So, should we use the first or second?

With different shapes, there is very nearly the same behavior at low Reynolds number, but at high Reynolds number there are different curves, roughly proportional to V^2. If use second, get one flat π_F in Stokes flow, multiple lines for different shapes. If use first, one line for laminar, multiple flats for different shapes. So the first is generally more convenient.

To make it physically relevant, we define f by $F = fKA$ so $f = F/KA$ where K is the kinetic energy density $\frac{1}{2}\rho V^2$ and A the relevant area, here the cross section $\frac{1}{4}\pi d^2$. And by convention we take the reciprocal of the above π_μ which is $\rho V d/\mu$. This gives: as the ordinate

$$f = f(\mathrm{Re}), \ \frac{F_d}{\frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2} = f\left(\frac{\rho V d}{\mu}\right). \tag{4.39}$$

Stokes flow (Re<0.1) friction factor:

$$F_d = 3\pi\mu d V \Rightarrow f = \frac{F_d}{KA} = \frac{3\pi\mu d V}{\frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2} = \frac{24\mu}{\rho V d} = \frac{24}{\mathrm{Re}}. \tag{4.40}$$

High Reynolds number friction factor:

$$F_d = c_d \cdot \frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2 \Rightarrow f = \frac{F_d}{KA} = c_d, \tag{4.41}$$

where c_d is the drag coefficient equal to 0.44 for a sphere.