March 30, 2005: Plate drag, turbulence

Recall from last time: Blassius solution to 2-D Navier Stokes around flat plate: u_x/U_∞ vs. $\beta = y\sqrt{U_\infty/\nu x}$; hits 0.99 at ordinate of 5. At $y = 0$, slope: 0.332, so viscous drag:

$$\tau_{yx} = -\mu\frac{\partial u_x}{\partial y} = -\mu\frac{\partial u_x}{\partial \beta}\frac{\partial \beta}{\partial y} \tag{4.58}$$

$$\tau_{yx} = -\mu \cdot 0.332 U_\infty \sqrt{\frac{U_\infty}{\nu x}} \tag{4.59}$$

Note: a function of x (larger near leading edge), diverges at $x = 0$! But $\delta \ll x$ does not hold there.

Now set to a friction factor:

$$\tau_{yx} = -0.332\sqrt{\frac{\rho\mu U_\infty^3}{x}} = f_x \cdot \frac{1}{2}\rho U_\infty^2 \tag{4.60}$$

This time τ is not constant, so we have different $f_x = \tau/K$ and $f_L = F_d/KA$. Let's evaluate both:

$$f_x = 0.664\sqrt{\frac{\mu}{\rho U_\infty x}} = \frac{0.664}{\sqrt{\mathrm{Re}_x}} \tag{4.61}$$

Lengthwise, global drag force, average friction factor. Neglect edge effects again...

$$F_d = \int \tau_{yx} dA = W\int_{x=0}^{L} tau_{yx} dx \tag{4.62}$$

$$F_d = W\int_{x=0}^{L} 0.332\sqrt{\frac{\rho\mu U_\infty^3}{x}}dx \tag{4.63}$$

$$F_d = 0.332W\sqrt{\rho\mu U_\infty^3} \cdot 2\sqrt{L} \tag{4.64}$$

$$F_d = 0.664W\sqrt{\rho\mu U_\infty^3 L} \tag{4.65}$$

Now for the average friction factor/drag coefficient:

$$f_L = \frac{F_d}{KA} = \frac{0.664W\sqrt{\rho\mu U_\infty^3 L}}{\frac{1}{2}\rho U_\infty^2 \cdot WL} = 1.328\sqrt{\frac{\mu}{\rho U_\infty L}} = \frac{1.328}{\sqrt{\mathrm{Re}_L}} \tag{4.66}$$

For a sphere, only defined average/global; for a tube, that and local are the same; but for a BL, they're different.

Also, note dimensionless BL thickness:

$$\delta = 5.0\sqrt{\frac{\nu x}{U_\infty}} \Rightarrow \frac{\delta}{x} = 5.0\sqrt{\frac{\nu}{U_\infty x}} = \frac{5.0}{\sqrt{\mathrm{Re}_x}} \tag{4.67}$$

Dimensionless entrance length too:

$$L_e = \frac{H^2 U_{av}}{100\nu} \Rightarrow \frac{Le}{H} = \frac{U_{av}H}{100\nu} = \frac{\mathrm{Re}_H}{100}. \tag{4.68}$$

Turbulence Started watching the movie, unfortunately didn't finish...