2.5 February 16, 2005: Wrapup radiation, dimensionless graphs

Anyone going to Chiang's lecture tomorrow night?

Muddy from last time:

- Why unstable for large Fo_M? Consider:
$$\frac{dT}{dt} = -T. \tag{2.38}$$
The solution is easy: $T = Ae^{-t}$. But let's try to solve it with finite differences, say $t = 0 \Rightarrow y = 1$. Using Δt=0.25 gives $T = 1, .75, .75^2, .75^3$... With Δt=1, we get $T = 1, 0, 0, 0$... With $\Delta t = 3$, we get $T = 1, -2, 4, -8, 16, ...$, which is clearly unstable.

- Please number equations on handouts... Are they not numbered?

Recap from last time: radiation is spontaneous emission, e_b etc. Defs: emissivity $\epsilon_\lambda = e_\lambda / e_{b,\lambda}$, the fraction of black body radiation which is emitted; absorptivity $\alpha_\lambda = a_\lambda / a_{b,\lambda}$. Cool result: $\epsilon_\lambda = \alpha_\lambda$, always! Material property. Graph resulting emission spectrum.

Fortunately e_b is quite simple:

$$e_b = \int_0^\infty e_{b,\lambda} d\lambda = \sigma T^4, \sigma = 5.67 \times 10^{-8} \frac{W}{m^2 \cdot K^4} \tag{2.39}$$

Grey body approximation: $\epsilon = \alpha = \epsilon_\lambda = \alpha_\lambda =$ constant. Makes life a lot simpler for us engineers. Superpose grey spectra on previous graph.

Resulting emission: $e = \epsilon \sigma T^4$. Pretty cool. Likewise average absorptivity α.

Averaged properties: $\epsilon = e/e_b$, $\alpha = a/incident$. Note ϵ will vary with temperature, α depends on wavelength of incident light. Example: global warming, CO_2 absorbs in the infrared, transmits sun in visible.

Little table:

	Wavelength	Total/average
BB Emission	$e_{b,\lambda}$	$e_b = \int_0^\infty e_{b,\lambda} d\lambda$
Actual emission	e_λ	$e(=q) = \int_0^\infty e_\lambda d\lambda$
Emissivity	$\epsilon_\lambda = e_\lambda / e_{b,\lambda}$	$\epsilon(T) = e/e_b$
Absorptivity	$\alpha_\lambda \equiv \epsilon_\lambda$	$\alpha(incident)$

That's as far as we'll go this year

Convective cooling curves Today's motivating example: Thermal spray. Small droplets, very rapid cooling, rapid solidification microstructures, solute trapping.

So, suppose initial condition $T = T_i$, outside fluid at T_{fl}. Boundary conditions: $r = R \Rightarrow q_r = h(T - T_{fl})$. Want to know temperature distribution through time, or temperature history. This requires a Bessel function series!! How to do understand?

- Dimensional analysis!
- Qualitative description of behavior.
- Graphs in text.
- Simpified low Biot number behavior: Newtonian cooling.

Dimensional analysis:

1. Formulation: $T - T_{fl} = f(t, r, R, T_i - T_{fl}, h, k, \rho c_p)$. $n = 8$ parameters!
2. Units: K, s, m, $\frac{\text{W}}{\text{m}^2 \cdot \text{K}}$, $\frac{\text{W}}{\text{m} \cdot \text{K}}$, $\frac{\text{J}}{\text{kg} \cdot \text{K}}$.

3. Base units: K, s, m, kg so $m = 4$.
4. Buckingham pi: four dimensionless parameters.
5. What to eliminate? Want to keep $T - T_{fl}$, t, r; choose h also. Eliminate R, $T_i - T_{fl}$, k, ρc_p.
6. π_T is easy, as is π_r. π_h: eliminated by k and R. π_t is funny, use k for seconds, ρc_p for Joules, R for remaining meters. Result is the Fourier number, the ratio of t/t_{SS}.

   Note: could have used h to eliminate seconds, but result wouldn't have been as cool: $\pi_t = ht/\rho c_p R$.
7. Dimensionless equation:

$$\frac{T - T_{fl}}{T_i - T_{fl}} = f\left(\frac{r}{R}, \frac{\alpha t}{R^2}, \frac{hR}{k}\right). \tag{2.40}$$