3.044 Recitation 7

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Topics

- Friction factors and drag forces
- Turbulence
- PS 5: Q&A

Friction Factors and Drag Forces for Different Geometries

=Flat Plate=

Laminar (Re < 10^5)

- Boundary layer thickness (δ: defined at where the velocity u_x is $0.99U_\infty$) is solved graphically. From Fig 4.4-3 (BSL[1]), $u_x/U_\infty$is 0.99 at $\beta = y\sqrt{U_\infty/\nu x}$=5.0. Thus

$$\delta = 5.0\sqrt{\frac{\nu x}{u_\infty}} = \frac{5.0x}{\sqrt{Re_x}} \ ,$$

where x is the position on the plate. This is valid only when we neglect the edge effect ($\delta \gg x$) .

- Friction factor (f_x :local friction factor and f_L :average friction factor)

$$f_x = \frac{\tau}{K} = \frac{-0.332\sqrt{\frac{\rho\mu U_\infty^3}{x}}}{\frac{1}{2}\rho U_\infty^2} = 0.664\sqrt{\frac{\mu}{\rho U_\infty x}} = \frac{0.664}{\sqrt{Re_x}}$$

$$f_L = \frac{F_d}{KA} = \frac{\int \tau_{yx} W dx}{KA} = \frac{0.664W\sqrt{\rho\mu U_\infty^3 L}}{\frac{1}{2}\rho U_\infty^2 WL} = \frac{1.328}{\sqrt{Re_L}}$$

where L is the length of the plate which is parallel to the flow and W is the width of the plate.

- Drag Force ($F_d = \int \tau_{yx} dA$):

$$F_d = 0.664W\sqrt{\rho\mu U_\infty^3 L}$$

Turbulent (Re > 10^5)

- Boundary layer

$$\delta = \frac{0.37x}{Re_x^{0.2}}$$

[1]R. B. Bird, W. E. Steward, and E. N. Lightfoot, *Transport Phenomena*, 2nd Ed., 2002.

- Friction factor

$$f_L = \frac{0.455}{(\log Re_L)^{2.58}} \; or \; f_L = \frac{0.146}{Re_L^{0.2}}$$

The first expression is taken from P&G[2]. The second expression is taken from BSL. They both are applicable to turbulent flow over a flat plate. For f_x,it is given [3] as

$$f_x = \frac{0.0576}{(Re_x)^{0.2}}$$

- Drag force (BSL)

$$F_d = 0.3 \sqrt[5]{\rho^4 \mu L^4 W^5 U_\infty^9}$$

=Parallel Plates=

Laminar (Re<1000)

- Boundary layer: same as flow over a plate
- Entrance length (L_e): Flow is fully-developed at $L > L_e$(at L_e, $\delta = H/2$ where H is the distance between the plates)

$$L_e = \frac{H^2 U_{av}}{100\nu}$$

=Tube=

Laminar (Re<2100)

- Friction factor: $f = \frac{\tau}{K} = \frac{8\mu U_{av}}{d} \cdot \frac{1}{\frac{1}{2}\rho U_{av}^2} = \frac{16}{Re}$
- Drag force (smooth surface) : $F_d = \frac{8\mu U_{av}}{d} \cdot \pi d L = 8\pi\mu U_{av} L$

Transitional - Turbulent (Re>2100)

- Friction factor: depends on surface roughness see plot of f vs. Re (Moody Plot) with different $\frac{\varepsilon}{d}$(roughness factor)
- Smooth pipe: $f = \frac{0.0791}{Re^{1/4}}$ (P&G)

=Sphere=

Stokes' Flow (Re<0.1)

- Friction factor: $f = \frac{24}{Re}$
- Drag force : $F_{d,} = 3\pi d \mu U_{av}$(valid for Re < 1)
- $Re \gg 1$: Friction factor $f = 0.44$

[2]D. R. Poier and G. H. Gieger, *Transport Phenomena in Materials Processing*, 1994.
[3]Welty, Wicks, Wilson, and Rorrer, *Fundamentals of Momentun Heat and Mass Transfer*, 4th Ed.

=Bubble=

- Friction factor: $f = \frac{16}{Re}$
- Drag force : $F_d = 2\pi d \mu U_{av}$

Introduction to Turbulence

eddy = a circular movement of fluid

λ= smallest eddy size (In the lecture note, this is wriiten as “l”), m

L= largest eddy size equals to the width of the flow, m

ε= energy dissipation, W/m^3

The smallest eddy Reynolds number is defined as

$$Re_\lambda = \frac{\rho u \lambda}{\mu} = 1 \rightarrow u = \frac{\mu}{\rho \lambda}$$

The largest eddy Reynolds number is defined as

$$Re_L = \frac{\rho u L}{\mu}$$

Energy dissipation in smallest eddies is

$$\varepsilon = \mu (\frac{\partial u}{\partial x})^2 \sim \mu \frac{u^2}{\lambda^2}$$

Substitute u with $\mu / \rho \lambda$ get

$$\varepsilon = \frac{\mu^3}{\rho^2 \lambda^4}$$

Solve for smallest eddy size in terns of ε

$$\lambda = \sqrt[4]{\frac{\mu^3}{\rho^2 \varepsilon}}$$

Example

Fluid with $\mu = 5 \times 10^{-3} \frac{Ns}{m^2}$ and $\rho = 1200 \frac{kg}{m^3}$ is flowing through a tube $d = 1cm$and $L = 30cm$ at $u_{av} = 0.5m/s$.

(a) What is the Q?

$Q = u_{av}A = 0.5 \times \frac{\pi}{4}(10^{-2}) = 3.93 \times 10^{-5}m^3/s$

(b) What is f?

$Re = \frac{\rho u_{av} d}{\mu} = \frac{(1.2\times10^3)(0.5)(10^{-2})}{5\times10^{-3}} = 1200$ (Laminar Re<2100)

$f = \frac{16}{Re} = 0.013$

(c) What is τ_{rz}?

The shear stress can be derived from u_{av} equation or can be estimated from the dimensionless shear stress ($f = \tau/K$)

$f = \frac{\tau}{K}$

$\tau = fK = 0.013 \times \frac{1}{2}\rho u_{av}^2 = 2\frac{N}{m^2}$

(d) What is F_d?

Since shear stress is constant, the drag force is

$F_d = \tau A = (\tau)(\pi dL) = 0.0189N$

$F_d = \frac{16\mu}{\rho u_{av} d}(\pi dL) = 8\pi\mu u_{av} L = 0.0189N$

It is shown that F_d in a tube flow does not depend on the tube diameter.

(e) What is pressure drop along the tube?

$\triangle P = \frac{F_d}{A} = \frac{0.0189}{\pi(0.01)^2/4} = 240\frac{N}{m^2} = 240$Pa

(f) If d is increased, how does it affect (a)-(e)? (assume same $u_{av}$and still in laminar regime).

$\uparrow d \quad \uparrow A, Q, Re, \downarrow f, \tau, \triangle P$

same F_d