3.044: Test 2 Practice Problems (Concepts & Calculations)

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1. Calculate pressure gradient needed for laminar-turbulent transition, (adapted from BSL 5A.1)
Fluid properties: viscosity $10^{-3} Ns/m^2$ density 1,320 kg/m^3
(a) Flow through a horizontal tube with diameter d=1.05 in.
(b) Flow through parallel plates with plate separation distance H=0.25 m

(a) $Re = \rho U_{av} d/\mu = 2100$ (Tube flow laminar criterion)
$U_{av} = 2100(10^{-3})/(1320 * 1.05 * 2.54 * 10^{-2})$=0.06 m/s
Tube flow average velocity
$U_{av} = \frac{\triangle PR^2}{8\mu L} \rightarrow \frac{\triangle P}{L} = \frac{8\mu U_{av}}{(d/2)^2} = 2.69\,\frac{N}{m^3}$

(b) $Re = \rho U_{av} H/\mu = 1000$ (Parallel plates lminar criterion)
$U_{av} = 1000(10^{-3})/(1320 * 1.05 * 2.54 * 10^{-2}) = .003\, m/s$
Parallel plate U_{av}
$U_{av} = \frac{\triangle PH^2}{3\mu L} \rightarrow \frac{\triangle P}{L} = \frac{3\mu U_{av}}{(H)^2} = 1.45 \times 10^{-4}\,\frac{N}{m^3}$

2. From PSET 4 No. 4, draw the shear stress distribution in the two fluid layers. If you don't know how to draw it qualitatively, you may proceed by doing calculation using the velocity profile given in the solution.

Using just one reference coordinate: z=0 at the plane
Shear at the top surface of glass layer is zero
At the interface, shear stress is negative. (Because du_x/dy is positve).
At the bottom surface, shear stress is negative.
The slope of shear stress with respect to z is smaller in glass compared to that in tin.

3. Estimate the entrance length for a laminar flow through a tube. (adapt from BSL 4A.7)

(a) Given that you know the U_{max}. The tube diameter is D. (Hint: approx the entrance length of flow in a tube using the same equation as that of flow over a flat plate).

(b) Rewrtie (a) in terms of (Re_D, D)

(a) Estimated boundary layer thickness: $\delta = 5.0\sqrt{\frac{\mu x}{\rho U_{av}}}$, $U_{av} = (1/2)U_{max}$

Entrance length is reached when δ approaches the radius length.

$\frac{D}{2} = 5.0\sqrt{\frac{2\mu L_e}{\rho U_{max}}}$

$L_e = \frac{\rho U_{max} D^2}{200\mu}$

(b) $Re = \rho U_{av} D/\mu = \rho U_{max} D/2\mu$

$L_e = \frac{Re_D D}{100}$

(same as that for flow through a parallel plate we derived in class except the characteristic length is the diameter.)

4. Two filter beds of Al_2O_3spheres are used to remove drossy oxides from the aluminum melt. The first shorter lenght bed on the left (A) where the flow enters has bigger particle size than the longer length bed on the right (B) where the flow exits. Given that $\omega_A = \omega_B$, L_A =0.7 L_B, and $d_A = 2d_B$. (adapted from P&G 3.13)

(a) Find the ratio of pressure drop through A to the pressure drop through B

(b) If the layout is in the vertical direction (i.e., A on the top and B on the bottom), write an expression of the pressure drop through each filter bed.

(a) Two packed bed in series automatically set the flow rate to be equal.

$Q_A = \frac{\mathcal{P}_{\mathcal{A}}}{\mu}\frac{\triangle P_A}{L_A}$=$Q_B = \frac{\mathcal{P}_{\mathcal{B}}}{\mu}\frac{\triangle P_B}{L_B}$

$\mathcal{P} = K1\frac{\omega^3}{s^2}$

$s_A = \frac{6}{d_A}(1-\omega)$ and $s_B = \frac{6}{d_B}(1-\omega)$

$\frac{\triangle P_A}{\triangle P_B} = \frac{L_A \mathcal{P}_{\mathcal{B}}}{L_B \mathcal{P}_{\mathcal{A}}} = \frac{L_A \lceil^{\in}_{\mathcal{B}}}{L_B \lceil^{\in}_{\mathcal{A}}} = (0.7)(4) = 2.8$

(b) If they are in vertical direction, the only change is $\triangle P' = \triangle P + \rho g L$

$\triangle P_A = \frac{Q\mu L_A}{K1\omega_A^3 d_A^2}(36)(1-\omega_A)^2 - \rho g L_A$

$\triangle P_B = \frac{Q\mu L_B}{K1\omega_B^3 d_A^2}(4)(36)(1-\omega_B)^2 - \frac{\rho g L_A}{0.7}$

** If the beds are in parallel, to get the same flow rate the permeability of A and B have to be equal. **

5. Aluminum oxide is solidified in a water cooled molybdynum molde to form continuous fibers with a diameter 200 μm.Estimate the length of the mold required to completely solidify the aluminum oxide as it exits from the mold, as a function of the fiber velocity. (P&G 10.11)

$Mo : h = 4000$W

$Al_2O_3 : k = 11W/m-K, C_p = 1230J/kg-K$, $\rho = 3016kg/m^3$, $T_M = 2327K$, $H_f = 1.07\times10^6 J/kg$

First, we should calculate Bi number to see what is the heat transfer limitting case.

$Bi = hD/k = (4000)(200\times10^{-6})/11 = 0.07$

It is seen that Bi<0.1. Thus heat transfer is limited by convection.

For solidification, the energy balance equation is

$h(T_m - T_s) = \rho C_p H_f u_{interface}$

In this problem, interface moves in r-direction while the fiber is being pulled in x-direction.

$h(T_m - T_s) = \rho C_p H_f \frac{dr}{dt}$

$r = \frac{h(T_m - T_s)}{\rho C_p H_f} t$

If we want the fiber to be completely solidified at the exit, it means that r approaches the fiber raduis at $x = L$. We can substitute $t = \frac{L}{u_x}$ and solve for L.

$\frac{D}{2} = \frac{h(T_m - T_s)L}{\rho C_p H_f u_x}$

$L = \frac{D\rho C_p H_f u_x}{h(T_m - T_s)}$