

Viscous Flow, Drag Force

3.044 March 16, 2005

Mechanics:

- Problem Set 4 due March 30
- Mid-term course evals today
- Notes and slides catch-up...

Today's lecture:

- Flow rate and average velocity
- Drag force on a sphere
- Reynolds number

Muddy from 3/14

Derivation error: generation includes radius

Flow rate, and related new topic: average velocity

Diffusive Phenomena

	Diffusion	Heat conduction	Fluid flow
What's conserved?	Moles of each species	Joules of energy	kg m/s momentum
Local density	C	$\rho c_p T$	$\rho \vec{u}$
Units of flux	$\frac{\text{mol}}{\text{m}^2 \cdot \text{s}}$	$\frac{\text{W}}{\text{m}^2}$	$\frac{\text{kg m}}{\text{m}^2 \cdot \text{s}} = \frac{\text{N}}{\text{m}^2}$
Conservation equation*	$\frac{\partial C}{\partial t} = -\nabla \cdot \vec{J} + G$	$\rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot \vec{q} + \dot{q}$	$\frac{\partial(\rho \vec{u})}{\partial t} = -\nabla P - \nabla \cdot \tau + \vec{F}$
Constitutive equation	$\vec{J} = -D \nabla C$	$\vec{q} = -k \nabla T$	$\tau = -\mu [\nabla \vec{u} + (\nabla \vec{u})^T]$
Diffusivity	D	$\alpha = k / \rho c_p$	$\nu = \mu / \rho$
Result**	$\frac{\partial C}{\partial t} = D \nabla^2 C + G$	$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{q}}{\rho c_p}$	$\frac{\partial \vec{u}}{\partial t} = -\frac{\nabla P}{\rho} + \nu \nabla^2 \vec{u} + \frac{\vec{F}}{\rho}$
Convective flux	$C \vec{u}$	$\rho c_p T \vec{u}$	$\rho \vec{u} \vec{u}$
New result**	$\frac{DC}{Dt} = D \nabla^2 C + G$	$\frac{DT}{Dt} = \alpha \nabla^2 T + \frac{\dot{q}}{\rho c_p}$	$\frac{D\vec{u}}{Dt} = -\frac{\nabla P}{\rho} + \mu \nabla^2 \vec{u} + \frac{\vec{F}}{\rho}$

*Only considering diffusive fluxes. T in fluid constit. means matrix transpose. **For uniform properties.

Stress conventions:

$$\sigma = -\tau - PI, \quad P = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$\tau_{xx} + \tau_{yy} + \tau_{zz} = 0, \quad \tau_{xy} = \tau_{yx} = -\sigma_{xy} = -\sigma_{yx}$$

Inclusions and Bubbles

Basic principle: drag force balances buoyancy force at terminal velocity

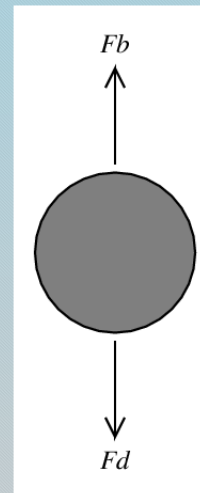
$$|\vec{F}_d| = |\vec{F}_b|$$

Buoyancy force is volume times density difference times gravity

$$F_b = \frac{1}{6} \pi d^3 (\rho_p - \rho_f) g$$

(For a bubble: $\rho_p = 0$)

Drag force is more complicated...

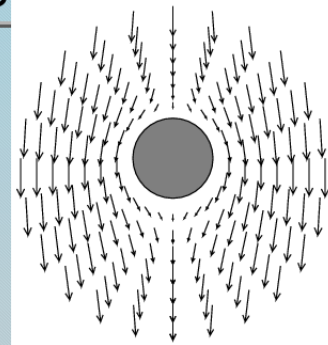


Inclusions and Bubbles: Stokes Flow

Fluid velocity around a solid sphere:

$$v_r = V \cos \theta \left[1 - \frac{3R}{2r} + \frac{1}{2} \left(\frac{R}{r} \right)^3 \right]$$

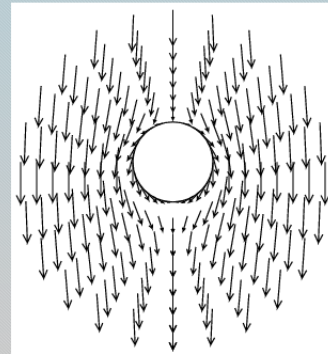
$$v_\theta = -V \sin \theta \left[1 - \frac{3R}{4r} - \frac{1}{4} \left(\frac{R}{r} \right)^3 \right]$$



Fluid velocity around a bubble:

$$v_r = V \cos \theta \left[1 - \frac{R}{r} \right]$$

$$v_\theta = -V \sin \theta \left[1 - \frac{1}{2} \frac{R}{r} \right]$$



Difference: fluid adheres to solid surface, bubble slips through with no shear traction

Inclusions and Bubbles: Drag Force

Roughly speaking: sum of viscous drag and kinetic energy terms

Sphere:

$$F_d = 3\pi d\eta V + 0.44 \cdot \frac{1}{2} \rho V^2 \cdot \frac{1}{4} \pi d^2$$

Bubble:

$$F_d = 2\pi d\eta V$$

Ratio of kinetic energy to viscous contribution: Reynolds number

$$\text{Re} = \frac{\rho V d}{\eta}$$

General friction factor:

$$F_d = f K A = f \cdot \frac{1}{2} \rho V^2 \cdot \frac{1}{4} \pi d^2$$

Solid extremes: $f = \frac{24}{\text{Re}} \Rightarrow f = c_d$

