Chapter 4

Fluid Dynamics

4.1 March 7, 2005: Intro, Newtonian Fluids

Fluid Dynamics! Brief introduction to rich topic, of which people spend lifetimes studying one small part. You will likely be confused at the end of this lecture, come to "get it" over the next two or three.

Categories: laminar, turbulent; confined (tubes and channels), free (jets, wakes); compressible, incompressible.

Outcomes: flow rates (define), drag force (integral of normal stress), mixing. Later couple with diffusion and heat conduction for convective heat and mass transfer.

Conservation of math (in one ear, out the other). But seriously, conservation of momentum.

Start: the 3.185 way. Momentum field, "momentum diffusion" tensor as shear stress. Show this using units: momentum per unit area per unit time:

$$\frac{\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}}}{\mathrm{m}^2\cdot\mathrm{s}} = \frac{\mathrm{kg}}{\mathrm{m}\cdot\mathrm{s}^2} = \frac{\mathrm{N}}{\mathrm{m}^2} \tag{4.1}$$

Two parallel plates, fluid between, zero and constant velocity. x-momentum diffusing in z-direction, call it τ_{zx}, one component of 2nd-rank tensor. Some conservation of math:

$$\text{accumulation} = \text{in} - \text{out} + \text{generation}$$

Talking about momentum per unit time, $\frac{\mathrm{kg}}{\mathrm{m}\cdot\mathrm{s}^2}$, locally momentum per unit volume $\rho\vec{u}$. Here suppose u_x varies only in the z-direction, no τ_{xx} or τ_{yx}, no u_y or u_z. Three conservation equations for three components of momentum vector, here look at x-momentum:

$$V\cdot\frac{\partial(\rho u_x)}{\partial t} = A\cdot\tau_{zx}\big|_z - A\cdot\tau_{zx}\big|_{z+\Delta z} + V\cdot F_x \tag{4.2}$$

Do this balance on a thin layer between the plates:

$$A\Delta z\frac{\partial(\rho u_x)}{\partial t} = A\tau_{zx}\big|_z - A\tau_{zx}\big|_{z+\Delta z} + A\Delta z F_x \tag{4.3}$$

Cancel A and divide by Δz, let go to zero:

$$\frac{\partial(\rho u_x)}{\partial t} = -\frac{\partial\tau_{zx}}{\partial z} + F_x \tag{4.4}$$

What's generation? Body force per unit volume, like gravity. Units: N/m^3 (like τ has N/m^2), e.g. ρg.

What's the constitutive equation for τ_{zx}? Newtonian fluid, proportional to velocity gradient:

$$\tau_{zx} = -\mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \tag{4.5}$$

This defines viscosity μ, which is the momentum diffusivity. Units: $\mathrm{N \cdot s/m^2}$ or $\mathrm{kg/m \cdot s}$, Poiseuille. CGS units: $\mathrm{g/cm \cdot s}$, Poise = 0.1 Poiseuille. Water: .01 Poise = .001 Poiseuille.

So, sub constitutive equation in the conservation equation, with $u_y = 0$:

$$\frac{\partial(\rho u_x)}{\partial t} = -\frac{\partial}{\partial z}\left(-\mu\frac{\partial u_x}{\partial z}\right) + F_x \tag{4.6}$$

With constant ρ and μ:

$$\rho\frac{\partial u_x}{\partial t} = \mu\frac{\partial^2 u_x}{\partial z^2} + F_x \tag{4.7}$$

It's a diffusion equation! Divide by ρ:

$$\frac{\partial u_x}{\partial t} = \nu\frac{\partial^2 u_x}{\partial z^2} + \frac{F_x}{\rho} \tag{4.8}$$

We have the diffusion equation! ν is the momentum diffusivity, like the thermal diffusivity $k/\rho c_p$ before it. Note: units of momentum diffusivity $\nu = \mu/\rho$: $\frac{\mathrm{kg/m \cdot s}}{\mathrm{kg/m^3}} = \mathrm{m^2/s}$! Kinematic ($\nu$), dynamic ($\mu$) viscosities.

So at steady state, with a bottom plate at rest and a top plate in motion in the x-direction at velocity U, we have: a linear profile, $u_x = Az + B$.

Note on graphics: velocities with arrows, flipping the graphs sideways to match orientation of the problem.

Case 1 today:

- Steady-state, no generation, bottom velocity zero, top U:

$$u_x = \frac{U}{L}z,\ \tau_{zx} = -\mu\frac{U}{L} \tag{4.9}$$

  Shear stress:

$$\tau_{zx} = -\mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) = -\mu\frac{U}{z} \tag{4.10}$$

  Thus the drag force is this times the area of the plate.