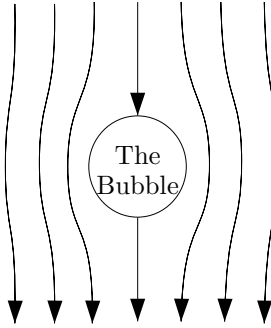


## 1. Floating Bubbles Out of Glass

- (a) The bubble is floating up, so in its frame of reference the fluid is flowing down:



- (b) This is just like terminal velocity of a solid sphere, but slightly simpler (since the weight is approximately zero) and with a different constant:

$$F_b = F_d = fKA,$$

$$\frac{1}{6}\pi d^3 \rho g = \frac{16\mu}{\rho U d} \cdot \frac{1}{2}\rho U^2 \cdot \frac{1}{4}\pi d^2,$$

$$U = \frac{\rho g d^2}{12\mu}.$$

- (c) The velocity which guarantees removal is the depth divided by the time. Solve for  $d$  and calculate, using that velocity:

$$d = \sqrt{\frac{12\mu U}{\rho g}} = \sqrt{\frac{12 \cdot 1.0 \frac{\text{kg}}{\text{m}\cdot\text{s}} \cdot \frac{0.1\text{m}}{60\text{s}}}{3200 \frac{\text{kg}}{\text{m}^3} \cdot 9.8 \frac{\text{m}}{\text{s}^2}}} = 8.0 \times 10^{-4}\text{m}.$$

- (d) Since  $V \propto d^2$ , if we cut  $d$  in half, the velocity will be one quarter of what it was, so it will take four times as long to float out the bubbles 0.4mm in diameter.
- (e) The larger bubbles will have larger  $d$  and larger  $U$  and thus a larger Reynolds number, so if they're Stokes then both are:

$$\text{Re} = \frac{\rho U d}{\mu} = \frac{3200 \frac{\text{kg}}{\text{m}^3} \cdot \frac{0.1\text{m}}{60\text{s}} \cdot 8.0 \times 10^{-4}\text{m}}{1.0 \frac{\text{kg}}{\text{m}\cdot\text{s}}} = 4.3 \times 10^{-3}.$$

This is easily within the Stokes flow régime.