2.2 February 7, 2005: More Conduction

Mechanics:

- Tests 3/9 and 4/20: in-lecture portion.
- Handouts: ABET, PS1 due Mon 2/14.
- Interesting lecture: ASM dinner Chiang "Ceramics in Electrochemical Systems" Thu Feb 17; students \$8, RSVP Sam Davis.

Spherical conduction with generation:

$$4\pi r^2 \Delta r \frac{\partial H}{\partial t} = 4\pi r^2 q_r\big|_r - 4\pi r^2 q_r\big|_{r+\Delta r} + 4\pi r^2 \Delta r \dot{q}$$

$$r^2 \rho c_p \frac{\partial T}{\partial t} = \frac{-r^2 k \frac{\partial T}{\partial r}\big|_r + r^2 k \frac{\partial T}{\partial r}\big|_{r+\Delta r}}{\Delta r} + r^2 \dot{q}$$

$$r^2 \frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{r^2 \dot{q}}{\rho c_p}$$

Steady-state solution:

$$A = \alpha r^2 \frac{\partial T}{\partial r} + \frac{\dot{q} r^3}{3\rho c_p}$$

$$T = \frac{A}{r} + B - \frac{\dot{q} r^2}{6\rho c_p}$$

Multi-layer solid Slides, mention heat transfer coefficient on outside as resistance $1/h$. Final result:

$$q = \frac{T_1 - T_{fl}}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h}}$$

Unsteady Solutions Responsible for two:

- 1-D semi-infinite uniform initial, constant T boundary:

$$\frac{T - T_s}{T_\infty - T_s} = \operatorname{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right). \quad (2.13)$$

  Example: two blocks of same material at different temperatures, come together (like a diffusion couple)

- 1-D infinite, uniform initial T, heat deposited at $x = 0$: Gaussian

$$T - T_i = \frac{(T_0 - T_i)\delta}{\sqrt{\pi \alpha t}} \exp\left(-\frac{x}{4\alpha t}\right). \quad (2.14)$$

  Example: resistance welding, brazing, some adhesives... Note $T_0\delta$ can be replaced with $H/\rho c_p$.

Even more on the handout, not responsible for any further than handout (and not asterisks either).