2.4 February 11, 2005: wrap up finite differences; radiation!

Explicit timestepping stability criterion:

$$\text{Fo}_M = \frac{\alpha \Delta t}{(\Delta x)^2} \leq \frac{1}{2} \quad \Rightarrow \quad \Delta t \leq \frac{1}{2}\frac{(\Delta x)^2}{\alpha} \tag{2.30}$$

Exercise: double spatial resolution, how much does timestep size change? Computational work?

2-D: $T_{i,j,n}$ at x_i, y_j, t_n: add another spatial derivative:

$$T_{i,j,n+1} = T_{i,j,n} + \frac{\alpha \Delta t}{(\Delta x)^2}(T_{i-1,j,n} - 2T_{i,j,n} + T_{i+1,j,n}) + \frac{\alpha \Delta t}{(\Delta y)^2}(T_{i,j-1,n} - 2T_{i,j,n} + T_{i,j+1,n}) + \frac{\dot{q}\Delta t}{\rho c_p} \tag{2.31}$$

For $\Delta x = \Delta y$:

$$T_{i,j,n+1} = (1 - 4\text{Fo}_M)T_{i,j,n} + 4\text{Fo}_M \frac{T_{i-1,j,n} + T_{i+1,j,n} + T_{i,j-1,n} + T_{i,j+1,n}}{4} + \frac{\dot{q}\Delta t}{\rho c_p} \tag{2.32}$$

Resulting stability criterion:

$$\text{Fo}_M = \frac{\alpha \Delta t}{(\Delta x)^2} \leq \frac{1}{4} \quad \Rightarrow \quad \Delta t \leq \frac{1}{4}\frac{(\Delta x)^2}{\alpha} \tag{2.33}$$

Handout: enthalpy method for phase changes, finite volume approach to BCs, timestepping approaches which avoid the stability criterion—but are much harder to implement.

Radiation! Def: spontaneous emission of photons from a hot body. Emission, absorption, reflection, transmission. Cosine distribution: hand-waving skin depth explanation.

Happens throughout a body, but surface emission follows a cosine distribution: handwaving explanation of skin depth as a function of angle.

Concept: black body, absorbs all incident radiation, theoretical construct with some practical application. Also emits maximum possible radiation. Handwaving explanation: zero reflection at the interface.

Defs: e is power emitted per unit area, e_b is power emitted by black body per unit area, e_λ is power per unit wavelength per unit area, $e_{b,\lambda}$ is power by black body per unit wavelength per unit area.

Emission spectrum of black body:

$$e_{b,\lambda} = \frac{2\pi h c^2 \lambda^{-5}}{e^{\frac{ch}{k_B \lambda T}} - 1} \tag{2.34}$$

h is Planck's constant, c is light speed, k_B Boltzmann's constant. Graph for different T.

Peak wavelength:

$$\lambda_{max} T = 2.9 \times 10^{-3} \text{m} \cdot \text{K} \tag{2.35}$$

1000K, 2.9μm=2900 nm; sun at 5800K is at 500 nm (yellow)—need to be pretty hot to peak in the visible spectrum.

How to get e_b? Integrate over all wavelengths. Fortunately, it's quite simple:

$$e_b = \int_0^\infty e_{b,\lambda} d\lambda = \sigma T^4 \tag{2.36}$$

The physicists must have jumped for joy when they saw that one. For our purposes, it puts radiation within reach of engineers. Okay, all done, never have to see that first equation again.

Even better:

$$\sigma = \frac{2\pi^5 k_B^4}{15 c^2 h^3} = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \tag{2.37}$$

Note: fourth-power dependence on temperature means this is **MUCH** more important at high temperature than low temperature.

New defs: emissivity $\epsilon_\lambda = e_\lambda / e_{b,\lambda}$, the fraction of black body radiation which is emitted; absorptivity $\alpha_\lambda = a_\lambda / a_{b,\lambda}$. Cool result: $\epsilon_\lambda = \alpha_\lambda$, always! Material property. Graph resulting emission spectrum.