1. Investment casting heat transfer[1]

   In the investment casting process, molten metal is poured into a ceramic shell mold, which we'll say is spherical, and allowed to solidify. If the temperature difference between the inside and outside of the mold is too large, thermal stress will cause the mold to fail and spill molten metal. (Such failures are often quite spectacular explosions, and can be very dangerous!)

   For this reason, the mold is pre-heated to near the melting point of the metal. However, while it cools due to convection and radiation, the outer surface of the mold may get significantly cooler than the inner surface, resulting in failure.

   Data:

   - mold outer radius $R = 10\text{cm}$
   - mold thickness $L = 1\text{cm}$
   - mold surface emissivity $\epsilon_m = 0.4$
   - mold thermal conductivity $k = 2\frac{\text{W}}{\text{m}\cdot\text{K}}$
   - Heat transfer coefficient $h = 100\frac{\text{W}}{\text{m}^2\cdot\text{K}}$

   (a) Express the net radiative heat flux from the mold surface as a function of mold surface temperature. You may assume the environment consists of black surfaces.

   (b) Calculate the steady-state temperature at the mold surface, by equating the conductive flux through the mold to the total radiative and convective flux from the surface, with an inner surface temperature of 1300K and a room temperature of 300K. (You may need to use an iterative method, such as Newton's method, to solve the final equation.)

   (c) Express the total radiative and convective flux as a constant times temperature $h_{\text{total}}T$, where h_{total} may be a function of temperature. This time you may use 0 for the environmental temperature in the convective component (this will somewhat overestimate the convective heat flux).

   (d) Use the h_{total} expression from part 1c and the thickness of the mold to estimate values of the Biot number for the mold at mold surface temperatures of 800K, 1100K and 1700K. Based on these Biot numbers, is the temperature likely to be uniform or non-uniform through the mold?

   (e) How different are your results from parts 1b and 1d?

   (f) If the temperature is too non-uniform, what design change might make it more uniform? (Assume you cannot make the mold any thinner for structural reasons.)

[1] Motivated by problems 6.11 and 11.11 8.14 in Poirier and Geiger, *Transport Phenomena in Materials Processing,* (Warrendale, PA: TMS), 1994.