

Analytical and Numerical Approaches to Heat Conduction Problems

3.044 February 9, 2005

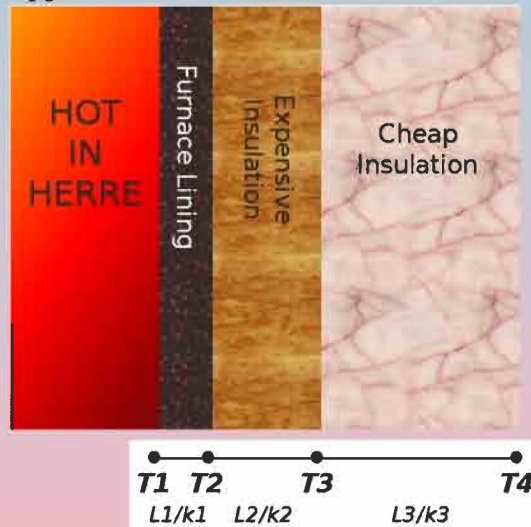
- Loose ends: multilayer cylinder, Biot number
- Gaussian solution to (thermal) diffusion equation
- Finite difference numerical solution of PDEs

Mechanics:

- Tests March 9, April 20: lecture portion
- Handout: Finite differences and heat conduction
- Lecture: ASM Dinner with Yet-Ming Chiang at the Faculty Club, Students \$8, RSVP Sam Davis

Multi-layered wall

Typical furnace wall:



Steady-state:

- Same heat flux everywhere
- Different conductivities, slopes
- Temperature is continuous (gaps...)
- Like current through multiple resistors in series
- "Voltage" = temperature
- "Resistance" = L/k
- "Current" = flux (or flux x area)

Multi-layered Cylindrical Wall

Multi-layer flat wall steady-state: heat flux is uniform

$$q_x = \frac{T_1 - T_{fl}}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h}}$$

Cylindrical wall: temperature in one layer

$$\frac{T - T_1}{T_2 - T_1} = \frac{\ln(r/R_1)}{\ln(R_2/R_1)}$$

Cylinder steady-state: flux-area product is uniform

$$Q = Aq_r = -2\pi r L k \frac{dT}{dr} = 2\pi r L k \frac{T_1 - T_2}{\ln(R_2/R_1)} \frac{1}{r}$$

Multi-layer cylinder: sum of resistances

$$Q = 2\pi r L q_r = \frac{2\pi L (T_1 - T_5)}{\frac{1}{k_1} \ln \frac{R_2}{R_1} + \frac{1}{k_2} \ln \frac{R_3}{R_2} + \frac{1}{k_3} \ln \frac{R_4}{R_3} + \frac{1}{R_4 h}}$$

Biot Number

Back to one layer and heat transfer coeff; flux:

$$q_x = \frac{T_1 - T_{fl}}{\frac{L}{k} + \frac{1}{h}}$$

The Biot number is defined as a ratio of resistances:

$$\text{Bi} = \frac{\text{solid conduction resistance}}{\text{fluid BL resistance}} = \frac{hL}{k}$$

Note: ratio of resistances is ratio of temperature drops:

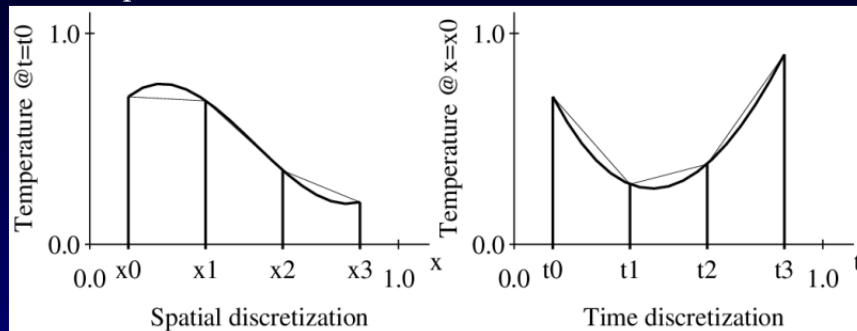
$$\text{Bi} = \frac{T_2 - T_1}{T_{fl} - T_2}$$

Next week: transient results using the Biot number

Finite Differences

When there's no (easy) analytical solution... use a computer!

Discretize space and time:



Then estimate derivatives

Finite Differences: instability

Instability limits maximum timestep size:

