1. Cooling of a little plastic widget

   Note that the ten seconds of free fall corresponds to 500 meters of height, so that part of the problem was just a bit unrealistic. Oh well.

   (a) The Biot number is simply

   $$\mathrm{Bi} = \frac{hL}{k} = \frac{40\frac{\mathrm{W}}{\mathrm{m}^2\cdot\mathrm{K}}\cdot 0.005\mathrm{m}}{2.0\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{K}}} = 0.1.$$

   The Newtonian Cooling assumption therefore applies, with uniform temperature across the widget.

   (b) The Fourier number is

   $$\mathrm{Fo} = \frac{\alpha t}{L^2} = \frac{kt}{\rho c_p L^2} = \frac{2.0\frac{\mathrm{W}}{\mathrm{m}\cdot\mathrm{K}}\cdot 10\mathrm{s}}{900\frac{\mathrm{kg}}{\mathrm{m}^3}\cdot 2500\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}\cdot(0.005\mathrm{m})^2} = 0.356.$$

   (c) Because the Biot number is so small, the Newtonian cooling equation should apply; this equation is very accurate even for complex geometries like this one.

   $$\frac{T - T_{fl}}{T_i - T_{fl}} = \exp\left(-\frac{Aht}{V\rho c_p}t\right)$$

   $$T = T_{fl} + (T_i - T_{fl})\exp\left(-\frac{Aht}{V\rho c_p}\right)$$

   $$T = 20^\circ\mathrm{C} + (160^\circ\mathrm{C} - 20^\circ\mathrm{C})\exp\left(-\frac{2\times 10^{-4}\mathrm{m}^2\cdot 40\frac{\mathrm{W}}{\mathrm{m}^2\cdot\mathrm{K}}\cdot 10\mathrm{s}}{5\times 10^{-8}\mathrm{m}^3\cdot 900\frac{\mathrm{kg}}{\mathrm{m}^3}\cdot 2500\frac{\mathrm{J}}{\mathrm{kg}\cdot\mathrm{K}}}\right)$$

   $$T = 20^\circ\mathrm{C} + 140^\circ\mathrm{C}\exp(-0.711) = 88.8^\circ\mathrm{C}$$

   This temperature is the uniform temperature of the whole widget, and thus applies in the "center" and everywhere else.

   (d) The thermal conductivity does not enter into the Newtonian cooling equation in part 1c, so it would appear that the final temperature would be unchanged.

   However, lowering the thermal conductivity increases the Biot number (since k is in the denominator), which in part 1a was shown to be just at the Newtonian cooling threshold. The final temperature in this case will thus be slightly higher than was predited in part 1c.

   Then again, a better estimate of the Biot number than in part 1a would use the volume/surface area ratio of 0.25 mm for L instead of the maximum dimension. In this case, the Biot number would go from 0.005 to 0.01, still raising the center temperature, but only very slightly.