1. Finite differences in heat conduction

   (a) If the properties are uniform, then the minimum number of explicit timesteps required is equal to the total time divided by the maximum timestep size. That timestep size is given by the mesh Fourier number, with the 2-D criterion for equal Δx and Δy given by:

$$\mathrm{Fo}_M = \frac{\alpha \Delta t}{(\Delta x)^2} \leq \frac{1}{4}$$

$$\Delta t \leq \frac{\Delta x^2}{4\alpha}$$

   The time to reach steady-state is simply:

$$t_{SS} = \frac{L^2}{\alpha}$$

   so the number of timesteps is:

$$\text{timesteps} = \frac{t_{SS}}{\Delta t} = \frac{L^2/\alpha}{(\Delta x)^2/4\alpha} = 4\left(\frac{L}{\Delta x}\right)^2$$

   Since $\Delta x = \Delta y = L/10$, at least 400 timesteps are required.

   (You could also have used $L/2$ as the lengthscale to calculate time to steady-state; in this case it would take 100 timesteps to reach steady state.)

   Note that this represents the time to conductive steady-state. The problem suggests that boundary conditions provide an extra resistance to heat conduction which is significant. If the effective Biot number is very large, then this is valid, but then for uniform properties, there are analytical solutions for this readily available which are not beyond your mathematical abilities. If the Biot number is very small, then the temperature is nearly uniform and one doesn't need this analysis; also, the larger resistance to heat transfer outside the cube would make for a much longer timescale. For intermediate Biot number situations, which most need finite differences, the time to steady state is likely not much longer than L^2/α, so that is of the right order of magnitude.

   (b) If we double the x- and y-resolution, then the grid points roughly quadruple.

   Since $\Delta t \propto (\Delta x)^2$, and Δx goes to half of what it was, Δt will be one quarter of what it was, and the number of timesteps will quadruple as well.

   The total computational work will increase by a factor of 16.

   (Actually, for 5×10 intervals, there are 6×11 grid points, which goes to 11×21, so there are really $21/6 = 3.5$ times more grid points than before, and the computational work will increase by a factor of 14. But this is close enough to 16 to make that a good estimate.)