3.044 Recitation 5

March 3-4, 2005

Topics

* PSET 2, 3
* Casting: Solid-Liquid Process
* Preparation for Test 1

Dimensional Analysis

Given the Gaussian Temperarture function as

$$T - T_{int} = \frac{\beta}{\rho C_p \sqrt{\pi \alpha t}} exp^{-\frac{x^2}{4\alpha t}}$$

1) All variables: $T - T_{int}, x, t, \alpha, (\beta/\rho C_p)$: n=5
2) Base units: $K, m, s, \frac{m^2}{s}, mK$:$K, m, s => T, L, t$: r=3
3) Number of dimensionless group: m=n-r=2
4) Keep: $\triangle T, x$: Eliminate: $t, \alpha, (\beta/\rho C_p)$
5) Form 2 π groups

$\pi_{\triangle T} = [T - T_{int}][t]^a[\alpha]^b[\beta/\rho C_p]^c$
$\pi_{\triangle T} = [T][t]^a[\frac{L^2}{t}]^b[LT]^c = [T]^{1+c}[t]^{a-b}[L]^{2b+c} = T^0 t^0 L^0 \; => c = -1, b = \frac{1}{2}, a = \frac{1}{2}$

$\pi_{\triangle T} = \frac{(T - T_{int})\rho C_p \sqrt{\alpha t}}{\beta}$

$\pi_x = [x][t]^a[\alpha]^b[\beta/\rho C_p]^c$
$\pi_x = [L][t]^a[\frac{L^2}{t}]^b[LT]^c = [T]^c[t]^{a-b}[L]^{1+2b+c} = [T]^0[t]^0[L]^0 \; => c = 0, b = -\frac{1}{2}, a = -\frac{1}{2}$

$\pi_x = \frac{x}{\sqrt{\alpha t}}$

$\pi_{\triangle T} = \frac{1}{\sqrt{\pi}} exp(-\frac{\pi_x^2}{4})$

Casting: Solid-Liquid Processes

In solidification process, the solid liquid interface moves with time (X(t)). Bi_s=hX(t)/k , so Bi_s changes with time. The solidification rate depends on the type of mold, metals being cast, and mold geometry. The net heat fluxes through the solid/liquid interface is the latent heat released during solidification.

$$\vec{q}_s \bullet \hat{n} - \vec{q}_l \bullet \hat{n} = -\rho \triangle H_M \frac{dX}{dt}$$

$\vec{q} \bullet \hat{n}$ is negative if the heat transfer direction is opposite to the normal vector of the interface. Therefore, if more heat is removed from solid than heat from liquid put into the interface, dX/dt is positive. Using this equation, we can derive the thickness of solidified metal (X) and solidfication rate (dX/dt) for different casting processes.

There are several types of molds used for metal casting. Two most common molds are metal mold and sand mold. Metal is a better heat conductor than sand. Therefore heat is transfered at faster rate in the metal mold than in the sand mold. Simplified case studies for each type of mold will be discussed.

Metal Mold (example in class)

- Good conductor
- Internal water cooling: surface conduction is dominant. $T_{mold} = T_{env}$
- Assume linear T profile through solid phase

Energy balance equation

Heat transfered at the mold surface = Heat conducted thru the solid phase =Latent heat during solidification

$$|q| = h(T_s - T_{env}) = \frac{k_s(T_m - T_{env})}{X}|_{S/L} = \rho \triangle H_M \frac{dX}{dt}|_{x=0}$$

At short time ($T_s = T_m$)

$$|q| = h(T_s - T_{env}) = \rho \triangle H_M \frac{dX}{dt}$$

$$X = \frac{h(T_m - T_{env})}{\rho \triangle H_M} t \quad \text{Linear growth (constant growth rate)}$$

At long time ($T_s = T_{env}$)

$$|q| = \frac{k_s(T_m - T_s)}{X} = \frac{k_s(T_m - T_{env})}{X} = \rho \triangle H_M \frac{dX}{dt}$$

$$\int_0^t \frac{k_s(T_m - T_{env})}{\rho \triangle H_M} dt = \int_0^X X\, dX$$

$$\frac{k_s(T_m - T_{env})}{\rho \triangle H_M} t|_0^t = \frac{X^2}{2}|_{X=0}^X$$

$$\frac{k_s(T_m - T_{env})}{\rho \triangle H_M} t = \frac{X^2}{2}$$

$$X = \sqrt{\frac{2k_s(T_m - T_{env})}{\rho \triangle H_M}} \sqrt{t}$$

Sand Mold

- Poor conductor
- Assume uniform T profile in solid $T_s = T_m$
- erf like T profile in the mold $T - T_m = (T_{env} - T_m) erf(\frac{X}{2\sqrt{\alpha t}})$

Energy balance equation

Heat transfered at the mold surface =Latent heat during solidification

$$|q| = k_{mold} \frac{dT}{dX}|_{x=0} = \frac{k_{mold}(T_m - T_{env})}{\sqrt{\pi \alpha_{mold} t}} = \rho \triangle H_M \frac{dX}{dt}$$

$$\int_0^t \frac{k_{mold}(T_m - T_{env})}{\rho \triangle H_M \sqrt{\pi \alpha_{mold} t}} dt = \int_0^X dX$$

$$\frac{k_{mold}(T_m - T_{env})}{\rho \triangle H_M \sqrt{\pi \alpha_{mold}}} 2t^{1/2}|_0^t = X|_{X=0}^X$$

$$X = \frac{2}{\sqrt{\pi}} \frac{k_{mold}(T_m - T_{env})}{\sqrt{\alpha_{mold}} \rho \triangle H_M} \sqrt{t}$$

Preparation for Test 1

Math

- -Vectors: Dot, Cross, Gradient, Divergence, Curl
- -erf and erfc function: graph, derivatives

Steady State: 1D Heat conduction ($0 = \alpha\nabla^2 T + \frac{\dot{q}}{\rho C_p}$)

- -Single layer: slab, cylinder, sphere
- -Multilayered wall: flat, cylinder, sphere
- -Heat flow Q=qA is constant in any shape
- -Approximate time to steady state for a thin body

Unsteady State: 1D Heat conduction ($\frac{\partial T}{\partial t} = \alpha\nabla^2 T + \frac{\dot{q}}{\rho C_p}$)

- -Unlimited heat source: erf
- -Limited heat source: Gaussian
- -Welding $\delta(T_0 - T_i) = H/(\rho C_p)$: H=total heat for thermal boundary layer. 2δ for infinite domain

Biot Number and Fourier Number

- - Small Bi: Uniform temperature in the body
- - Large Bi: Large temperature gradient in the body
- -How to apply Fo and Bi in practice

Finite Difference

- -Explicit timestepping method
- -Mesh Fourier number
- -Maximum $\triangle t$

Dimensional Analysis

- -Represent some physical features
- -Dimensionless form of equation
- -Buckingham Pi Theorem

Radiation

- -Black body is a perfect emitter.
- - Emissivity of any body will be a fraction of black body emissivity
- -Unsteady state heat tansfer by radiation

Economic Models in Engineering

- -Present value, Future value, Equal payment
- -NPV and Annual payment
- -Product volume and Effective Production Volume
- -Dedicated equipment cost
- -Non-Dedicated equipment cost
- -Material cost