1. Turbulence and mixing in a tube

   Data: water density $\rho = 1000 \frac{\text{kg}}{\text{m}^3}$, viscosity $\eta = 0.001 \frac{\text{kg}}{\text{m}\cdot\text{s}}$.

   (a) Sketch the (time-smoothed) velocity profiles for laminar and turbulent flow in a tube.

   (b) Show that the pressure drop (pressure difference between the entrance and exit) required to drive water flow at a rate of 0.001 m^3/s through a tube 30 m long and 1 cm (0.01 m) diameter with inner surface roughness of approximately 10 microns (10^{-5} m) is approximately 5.3 MPa ($5.3 \times 10^6 \frac{\text{N}}{\text{m}^2}$).

   (c) Using the Hägen-Poiseuille equation, estimate the effective turbulent viscosity, that is, the viscosity which would give the pressure drop from part 1b for this flow rate if flow were laminar.

   (d) A (water-soluble) macromoleculular substance is introduced into the flow near the entrance of the tube. Using a turbulent Prandtl number of one, estimate the turbulent diffusivity of the macromolecule from the turbulent viscosity in part 1c. How much time does it take to achieve uniform mixing of the substance across the tube?

   (e) Given that power required to drive this flow is flow rate times pressure drop (power = $Q\Delta P$), estimate the average energy dissipation rate per unit volume in the tube under conditions described in part 1b.

   (f) Estimate the turbulent microscale, the size of the smallest eddies, in this tube.

   (g) If the diffusivity of the macromoleculular substance in water is $D = 10^{-6}\text{cm}^2/\text{s}$ ($= 10^{-10}\text{m}^2/\text{s}$), estimate the time required for total homogenization of the mixture.

   (h) Estimate the pressure drop required to cut the homogenization time (and tube length) in half.