April 1, 2005: Turbulence

Turbulence Starting instability, energy cascade. Vortices grow in a velocity gradient because of momentum convection, damped due to viscosity; therefore, tendency increases with increasing Re.

Resulting behavior:

- Disorder.
- "Vorticity" in flow, 3-D.
- Lots of mixing, of mass and heat as well as momentum.
- Increased drag due to momentum mixing, as small vortices steal energy from the flow.

Turbulent transport: enhanced viscosity, also enhanced diffusivity, conductivity. $\nu_t \simeq \alpha_t \simeq D_t$.

Energy cascade and the Kolmogorov microscale. Largest eddy Re=UL/ν, smallest eddy Reynolds number $u\ell/\nu \sim 1$. Energy dissipation, W/m^3; in smallest eddies:

$$\epsilon = \eta \left(\frac{du}{dx}\right)^2 \sim \eta \frac{u^2}{\ell^2}. \tag{4.69}$$

Assuming most energy dissipation happens there, we can solve these two equations, get smallest eddy size and velocity from viscosity, density and dissipation:

$$u \sim \ell \sqrt{\frac{\epsilon}{\eta}} \Rightarrow \frac{\rho \ell^2}{\eta} \sqrt{\frac{\epsilon}{\eta}} \sim 1, \tag{4.70}$$

$$\ell \sim \left(\frac{\eta^3}{\rho^2 \epsilon}\right)^{\frac{1}{4}}. \tag{4.71}$$

This defines the turbulent microscale. For thermal or diffusive mixing, turbulence can mix things down to this scale, then molecular diffusion or heat conduction has to do the rest. Time to diffusive mixing in turbulence is approximately this ℓ^2/D.

So suppose we turn off the power, then what happens? Smallest eddies go away fast, then larger ones, until the whole flow stops. Timescale of smallest is ℓ^2/ν, largest is L^2/ν_t, turbulent effective viscosity. Get into modeling and structure later if time is available.

[Didn't cover...] Modeling: $K - \ell$ and $K - \epsilon$ modeling (C_μ, C_1, C_2, σ_K and σ_ϵ are empirical constants):

$$K = \frac{1}{2}\rho(u_x'^2 + u_y'^2 + u_z'^2), \nu_t = C_\mu \frac{K^2}{\epsilon}. \tag{4.72}$$

$$\frac{DK}{Dt} = \nabla \cdot \left(\frac{\nu_t}{\sigma_K} \nabla K\right) + \nu_t \nabla \vec{u} \cdot \left(\nabla \vec{u} + (\nabla \vec{u})^T\right) - \epsilon. \tag{4.73}$$

$$\frac{D\epsilon}{Dt} = \nabla \cdot \left(\frac{\nu_t}{\sigma_\epsilon} \nabla \epsilon\right) + C_1 \frac{\nu_t \epsilon}{K} \nabla \vec{u} \cdot (\nabla \vec{u} + (\nabla \vec{u})^T) - C_2 \frac{\epsilon^2}{K}. \tag{4.74}$$

Parviz Moin and John Kim, "Tackling Turbulence with Supercomputers," *Scientific American* January 1997 pp. 62-68.

> Turbulence may have gotten its bad reputation because dealing with it mathematically is one of the most notoriously thorny problems of classical physics. For a phenomenon that is literally ubiquitous, remarkably little of a quantitative nature is known about it. Richard Feynman, the great Nobel Prize-winning physicist, called turbulence "the most important problem of classical physics." Its difficulty was wittily expressed in 1932 by the British physicist Horace Lamb, who, in an address to the British Association for the Advancement of Science, reportedly said, "I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic."