3.2 March 2, 2005: Moving Body, Stability

Recall: vacuum arc remelting. Setup, geometry, frame of reference, relative z-velocity $u_z < 0$. Convective flux $\vec{q}$="$H\vec{u}$" or $\rho c_p \vec{u}$, Back up a couple of steps:

$$\rho c_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2} - \rho c_p u_z \frac{\partial T}{\partial z} + \dot{q} \tag{3.9}$$

Cancel ρc_p:

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial z^2} - u_z \frac{\partial T}{\partial z} + \frac{\dot{q}}{\rho c_p} \tag{3.10}$$

Go through terms: regular accumulation, regular second derivative with thermal diffusivity. Next the convective term, discuss in terms of units and derivatives. Rewrite again in terms of substantial derivative:

$$\frac{DT}{Dt} = \alpha\frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{\rho c_p} \tag{3.11}$$

Steady-state, no generation: temperature depends only on z, not t:

$$\alpha\frac{d^2 T}{dz^2} - u_z\frac{dT}{dz} = 0 \tag{3.12}$$

Simple solution using the characteristic polynomial, $R = 0, u_z/\alpha$. Result:

$$T = A + B\exp\left(\frac{u_z z}{\alpha}\right) \tag{3.13}$$

Fit to boundary conditions: $z = 0 \Rightarrow T = T_M$, $z = \infty \Rightarrow T = T_i$ so use erf-style:

$$\frac{T - T_i}{T_M - T_i} = \exp\left(\frac{u_z z}{\alpha}\right) \tag{3.14}$$

Lengthscale=α/u_z. Graph, noting that u_z is negative. Titanium $\alpha = 0.1\frac{\text{cm}^2}{\text{s}}$, $u_z \sim 5\frac{\text{cm}}{\text{min}} = \frac{1}{12}\frac{\text{cm}}{s}$, so $\alpha/u_z = 1.2cm$, about 1/2 inch. So only the bottom few centimeters are heated at all, even at this low velocity!

Heat flux into the bottom:

$$q_z = -k\frac{\partial T}{\partial z} = -k(T_m - T_i)\frac{u_z}{\alpha}\exp\left(\frac{u_z z}{\alpha}\right) = -\rho c_p u_z (T_m - T_i) \tag{3.15}$$

Note $\rho c_p(T_m - T_i)$ is the enthalpy per unit volume to heat metal to its melting point. Mult by u_z for enthalpy per unit area to heat metal coming at a rate of u_z, which is a cool result.

This is heat flux into the solid. What about into the liquid? Remember last time:

$$\vec{q}_s \cdot \hat{n} - \vec{q}_\ell \cdot \hat{n} = -\rho \Delta H_M \frac{dX}{dt} \tag{3.16}$$

Replace dX/dt with u_z and we're done, need to supply the heat of melting and of getting to this temperature. Here larger $\vec{q}_\ell$ than $\vec{q}_s$, both positive, so dX/dt positive.

Stability Is the VAR melt front stable? What if we have a bump, or groove?

Solidification: is the growing solid shell stable?

What about a solid particle growing into an undercooled liquid?

What about alloy solidification?