Viscous Flow, Drag Force

3.044 March 14, 2005

Mechanics:

- Problem Set 4 due March 28 (or 30?)
- Mid-term course evals Wednesday
- Notes and slides catch-up...

Today's lecture:

- Pressure-driven flow in a tube
- Recap: diffusion, conduction, viscous flow
- Drag force on a sphere

Diffusive Phenomena

	Diffusion	Heat conduction	Fluid flow
What's conserved?	Moles of each species	Joules of energy	kg m/s momentum
Local density	C	$\rho c_p T$	$\rho\vec{u}$
Units of flux	$\frac{\text{mol}}{\text{m}^2\cdot\text{s}}$	$\frac{\text{W}}{\text{m}^2}$	$\frac{\text{kg}\frac{\text{m}}{\text{s}}}{\text{m}^2\cdot\text{s}} = \frac{\text{N}}{\text{m}^2}$
Conservation equation*	$\frac{\partial C}{\partial t} = -\nabla\cdot\vec{J} + G$	$\rho c_p \frac{\partial T}{\partial t} = -\nabla\cdot\vec{q} + \dot{q}$	$\frac{\partial(\rho\vec{u})}{\partial t} = -\nabla P - \nabla\cdot\tau + \vec{F}$
Constitutive equation	$\vec{J} = -D\nabla C$	$\vec{q} = -k\nabla T$	$\tau = -\mu\left[\nabla\vec{u} + (\nabla\vec{u})^T\right]$
Diffusivity	D	$\alpha = k/\rho c_p$	$\nu = \mu/\rho$
Result**	$\frac{\partial C}{\partial t} = D\nabla^2 C + G$	$\frac{\partial T}{\partial t} = \alpha\nabla^2 T + \frac{\dot{q}}{\rho c_p}$	$\frac{\partial\vec{u}}{\partial t} = -\frac{\nabla P}{\rho} + \nu\nabla^2\vec{u} + \frac{\vec{F}}{\rho}$
Convective flux	$C\vec{u}$	$\rho c_p T\vec{u}$	$\rho\vec{u}\vec{u}$
New result**	$\frac{DC}{Dt} = D\nabla^2 C + G$	$\frac{DT}{Dt} = \alpha\nabla^2 T + \frac{\dot{q}}{\rho c_p}$	$\frac{D\vec{u}}{Dt} = -\frac{\nabla P}{\rho} + \mu\nabla^2\vec{u} + \frac{\vec{F}}{\rho}$

*Only considering diffusive fluxes. ^{T} in fluid constit. means matrix transpose. **For uniform properties.

Stress conventions:

$$\sigma = -\tau - PI,\ P = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$\tau_{xx} + \tau_{yy} + \tau_{zz} = 0,\ \tau_{xy} = \tau_{yx} = -\sigma_{xy} = -\sigma_{yx}$$

Inclusions and Bubbles

Basic principle: drag force balances buoyancy force at terminal velocity

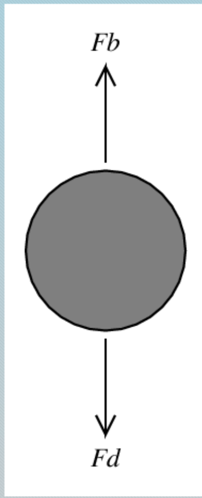
$$|\vec{F}_d| = |\vec{F}_b|$$

Buoyancy force is volume times density difference times gravity

$$F_b = \frac{1}{6}\pi d^3(\rho_p - \rho_f)g$$

(For a bubble: $\rho_p = 0$)

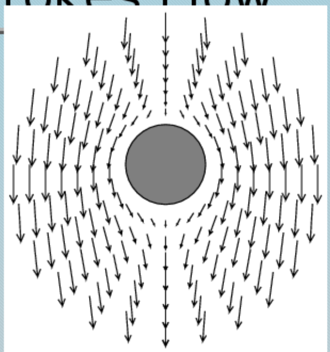
Drag force is more complicated...



Inclusions and Bubbles: Stokes Flow

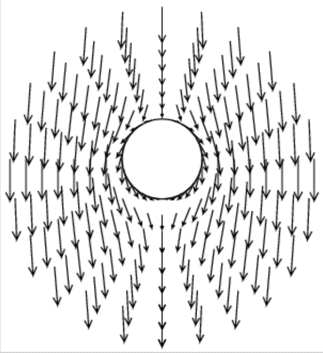
Fluid velocity around a solid sphere:

$$\begin{aligned} v_r &= V\cos\theta\left[1 - \frac{3}{2}\frac{R}{r} + \frac{1}{2}\left(\frac{R}{r}\right)^3\right] \\ v_\theta &= -V\sin\theta\left[1 - \frac{3}{4}\frac{R}{r} - \frac{1}{4}\left(\frac{R}{r}\right)^3\right] \end{aligned}$$



Fluid velocity around a bubble:

$$\begin{aligned} v_r &= V\cos\theta\left[1 - \frac{R}{r}\right] \\ v_\theta &= -V\sin\theta\left[1 - \frac{1}{2}\frac{R}{r}\right] \end{aligned}$$



Difference: fluid adheres to solid surface, bubble slips through with no shear traction

Inclusions and Bubbles: Drag Force

Roughly speaking: sum of viscous drag and kinetic energy terms

Sphere:

$$F_d = 3\pi d\eta V + 0.44 \cdot \frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2$$

Bubble:

$$F_d = 2\pi d\eta V$$

Ratio of kinetic energy to viscous contribution: Reynolds number

$$\mathrm{Re} = \frac{\rho V d}{\eta}$$

General friction factor:

$$F_d = fKA = f \cdot \frac{1}{2}\rho V^2 \cdot \frac{1}{4}\pi d^2$$

Solid extremes: $f = \dfrac{24}{\mathrm{Re}} \Rightarrow f = c_d$