April 11, 2005: Non-Newtonian flow

Recall 2-D channel flow at steady-state:

$$0 = -\frac{\partial P}{\partial x} - \frac{\partial \tau_{xx}}{\partial x} - \frac{\partial \tau_y x}{\partial y} + F_x. \quad (4.111)$$

In the absence of force in the x-direction, and with uniform pressure gradient everywhere $\partial P/\partial x = \Delta P/L$, we get:

$$\tau_{yx} = \frac{\Delta P}{L} y + const. \quad (4.112)$$

Shear stress is linear regardless of the flow rheology.

Non-Newtonian: graphs of τ_{yx} vs. $\partial u_x/\partial y$. Categories:

- Bingham plastic: finite yield stress, beyond that moves with strain proportional to stress minus yield stress, but up to it nothing. Some heavily-loaded liquids, polymer composites; semi-solid metals, toothpaste bond together then break free above τ_y.

  Model: yield stress τ_y, slope μ_P:

  $$ga\dot{m}ma = \begin{cases} 0, |\tau| < \tau_y \\ \frac{\tau - \tau_y}{\mu_P}, |\tau| > \tau_y \end{cases} \quad (4.113)$$

  Result: material in the center moves together uniformly, shear layers on either side. If not enough $\Delta P/L$ then it doesn't move at all.

  More than 1-D leads to wierd Tresca, von Mises yield criteria, etc.

- Pseudoplastic (shear-thinning), examples: heavily-loaded semi-solid, many polymers get oriented then shear more easily.

  Model: power law, $n < 1$.

  Power law relation:

  $$\tau_{yx} = \mu_0 \left(\frac{\partial u_x}{\partial y}\right)^n$$

- Dilatant (shear-thickening), example: fluid with high-aspect ratio solid bits; blood. More mixing, momentum mixing, acts like viscosity. Platelet diffusivity, concentration near walls...

  Model: power-law, $n > 1$.

Viscoelasticity Frictional damping in an elastic solid. Result: slow time-dependent springback. Especially important in polymers; in metals leads to springback after deformation.

Mechanism: example of loading in polymers:

- When first loaded, can have rapid elastic response due to bond stretching and bending.
- Over time, the polymer slowly (depending on temperature) flows to accomodate stress, orienting its molecules in the process.
- When released, the elastic response is restored right away.
- Then entropy slowly flows the molecules into their original isotropic random (non-aligned) conformation, returning the body to its original shape.

It's "elastic" because the final shape is the same as the initial, "viscous" because the change happens slowly, hence "visco-elastic".

Model: spring in series with (spring and dashpot in parallel). More complex: take 3.064 Polymer Engineering.