

3.044 Recitation 4

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Topics

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Cash Flow Analysis

NPV = Net Present Value = Sum of all cash flows over time towards the present value

Annual Equivalent = Set of equal payments calculated towards present value

A = Payment

P = Present Value

r = Discounted Rate

F = Future value

caf = compound amount factor

crf = capital recovery factor

N = Period (Number of payments i.e. 10 months of monthly payment, 5 years of annual payment)

Single Payment	Finite Series of Equal Payments
$F = P(1+r)^N$	$F = \sum_i^N A(1+r)^i = A \frac{[(1+r)^N - 1]}{r}$ $A = Pr \frac{(1+r)^N}{[(1+r)^N - 1]}$
$NPV = \sum_i^N \frac{F}{(1+r)^i}$	$AE = (NPV) \frac{r(1+r)^N}{(1+r)^N - 1}$
Small period	Infinite Series of Equal Payments
$(1+r)^N = e^{rN}$	$F = \sum_i^N Pr(1+r)^i$

Table 1. Cash Flow Analysis

Excel has a built-in function to calculate NPV and Annual Equivalent Payment (named PMT in Excel). The syntax are as follow:

NPV = Initial cost + NPV(r, F1, F2, ...)

Annual Equivalent = PMT(r, N, -P)

Example

1. You are a project manager. Your task is to choose between the two technology options that would yield the least expense over 3 years.

Option 1: A cutting machine which uses a normal blade that requires changing of the blade more frequently.

Option 2: A cutting machine which uses a diamond coated blade which has longer service life.

Parameters	Option 1	Option 2
r=10%		
Purchase cost	2000	3500
Maintenance cost per year (M)	300	50

Method A: Solve by Finance Model equations given in Table 1.

Step 1. Must understand that the given value for maintenance cost is in terms of future cost (F).

Step 2. First convert the future cost of maintenance to present value

$$P = \frac{F}{(1+r)^t}, \text{ where } i=1, 2, 3$$

Year (N)	Option 1	Option 2
1	$\frac{300}{(1+0.10)^1} = 272.73$	$\frac{50}{(1+0.10)^1} = 45.45$
2	$\frac{300}{(1+0.10)^2} = 247.93$	41.32
3	$\frac{300}{(1+0.10)^3} = 225.39$	37.57
Sum Maintenance cost	746	124
Total Expense	2000 + 746 = 2746	3500 + 124 = 3624

Step 3: You can also calculate Annual Equivalent Payment

$$AE = \frac{(NPV)r(1+r)^N}{(1+r)^N - 1}, \text{ where } N = 3$$

Option 1: Annual Equivalent = $\frac{(2746)(0.10)(1+0.10)^3}{(1+0.10)^3 - 1} = 1104$

Option 2: Annual Equivalent = $\frac{(3624)(0.10)(1+0.10)^3}{(1+0.10)^3 - 1} = 1457$

Step 4: By comparing the net present values or annual equivalent cost, you will choose Option 1.

Method B: Use Excel spreadsheet

Step 1: Must know that we need to calculate NPV. Cost are Purchase cost + Present values of maintenance cost

Step 2: Let NPV and PMT functions do the calculation

NPV=Purchase cost + NPV(r, M1, M2, M3)

Option 1: NPV = 2000 + NPV(0.10, 300, 300, 300) = 2746

Option 2: NPV = 3500 + NPV(0.10, 50, 50, 50) = 3624

PMT=PMT(r, N, -P)

Option 1: PMT=PMT(0.10, 3, -2746)=1104

Option 2: PMT=PMT(0.10, 3, -3624)=1457