18.022 Hour Test
September 30, 2005
CLOSED BOOK; NO BOOKS, NOTES, OR CALCULATORS

Name Rec.Instr. Rec.Time

Please show all your work on this paper, and identify your answers clearly. Use backs of pages if necessary. Points for each question are as shown (for a total of 100 points). If you are stalled on one question, go on to the next.

1. (10) Simplify each of the following (use laws of vector algebra, not coordinate formulas.)

(a) $(2\vec{A}-\vec{B})\cdot(\vec{A}+2\vec{B})$

(b) $(2\vec{A}-\vec{B})\times(\vec{A}+2\vec{B})$

(c) $\vec{A}\cdot((\vec{B}\times\vec{C})\times(\vec{C}\times\vec{A}))$

SCORE: 1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

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2. (20) Let M be the plane $x + y + z = 0$. Let L be the line given by $\vec{R}(t)=\hat{i}+t\,\hat{j}\ (-\infty<t<\infty)$.

(a) Find the intersection P of L with M.

(b) Find a parametric representation for the line L_1 which lies in M, goes through P, and is perpendicular to L.

(c) Find a parametric representation for the line L_2 which is the projection of L in M.

3. (20) A particle leaves the origin at time t = 0 with initial velocity $v_o = \hat{i} + 2\hat{j}$. For all t ≥ 0, its acceleration is given by $\vec{a}(t) = 2\cos t\,\hat{i} - 2\sin t\,\hat{j}$. Find the position $\vec{R}(t)$ and velocity $\vec{v}(t)$ of the particle at any time $t \geq 0$.

4. (20) You are given the following information about vectors $\vec{A}, \vec{B}$, and $\vec{C}$.

$$(\vec{A}\times\vec{B})\times\vec{A} = \vec{0}$$

$$\vec{B}\cdot\vec{B} = 4$$

$$\vec{A}\cdot\vec{B} = -6$$

$$\vec{B}\cdot\vec{C} = 6\ .$$

Find the following values:

(a) $\vec{A}\times\vec{B} =$

(b) $\vec{A}\cdot\vec{A} =$

(c) $\vec{A}\cdot\vec{C} =$

(d) $\vec{A}\cdot(\vec{B}\times\vec{C}) =$

(Hint: Easiest if you don't use coordinates.)

5. (20) Let $f(x,y,z) = x^2 + xy + yz$, and let P be the point $P = (2, -1, -2)$.

(a) Find the gradient of f at P.

(b) Find the directional derivative of f at P in the direction of the origin.

(c) Let Q be the point at distance 0.03 from P in the direction of the origin. Estimate the value of f at Q.

(d) Find the equation of the tangent plane at P to the surface $x^2 + xy + yz = 4$.

(Hint: In (d), use the fact that the normal direction to the tangent plane at P to a level surface of a function f is given by the direction of the gradient of f at P.)

6. (10) (a) Let M be a plane and let L be a line. Must it always be true that there is at least one plane M* such that L lies in M* and M* is perpendicular to M?

(b) Is it always true that there is exactly one such plane M*? Explain your answer.