

18.002 – Fall 2005
SOME SAMPLE QUESTIONS FOR HOUR TEST 2
ANSWERS (2nd version)

(1) At a given point P, a scalar field f has the following directional derivatives:

$$df/ds = 3 \text{ in the direction } \hat{i},$$

$$df/ds = -2 \text{ in the direction } \hat{j},$$

$$df/ds = -\sqrt{3} \text{ in the direction of } \hat{i} + \hat{j} - \hat{k}.$$

At the point P, what is the directional derivative of f in the direction of $2\hat{i} + 2\hat{j} - \hat{k}$.

(Remember to use unit vectors when unit vectors are called for.) **-2/3**

(2) The following equation defines z implicitly as a function (multivalued) of x and y:

$$xz^2 + yz = 3.$$

Find all possible values of $\left(\frac{\partial z}{\partial x}\right)_y$ when $x = 1$ and $y = 2$. **1/4, -9/4 (3)**

What are the global maximum and minimum values attained by $f(x,y,z) = xyz$ on the ellipsoid surface $2x^2 + 2y^2 + z^2 = 2$? **Max: $\sqrt{6}/9$; Min: $-\sqrt{6}/9$.**

(4) Consider the triple integral $\iiint_R xyz \, dV$, where R is that portion of the solid sphere of radius 2 with center at the origin which lies above the plane $z = 0$ and on the $x \geq 0$ side of the plane $x = 0$.

(a) By definition, evaluate this integral. **0**

(b) Express this integral as an iterated integral in spherical coordinates.

$$\int_{-\pi/2}^{\pi/2} \int_0^{\pi/2} \int_0^2 \rho^5 \sin^3 \phi \cos \phi \sin \theta \cos \theta \, d\rho \, d\phi \, d\theta$$

(5) Let R be the region in the first quadrant contained between the lines $y = x$ and $y = 2x$ and also between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(a) (5) Sketch the region R.

(b) (15) Let $f(x,y) = (x^2 + y^2)/x^2$. Use a change of variables with $u = x^2 + y^2$ and $v = y/x$ to find $\iint_R f \, dA$. **3/2**

(Hint. Note that $f(x,y) = 1 + (y/x)^2$.)