MATHEMATICAL OBJECTS

MATHEMATICAL WORLD vs. PHYSICAL UNIVERSE

Math world: numbers, functions, operations, geometries, infinities, limits.

Existence of math objects.

Creation of math objects:
- -from sets
- -by definition from existing math objects

"Ideal" and perfect nature of math objects

Study and exploration of math objects and their properties by deduction ("proof"). Assumptions and theorems (= facts)

Remarkable agreement of community of scientists and mathematicians as to correctness of mathematical investigations and results

2-DIMENSIONAL EUCLIDEAN GEOMETRY (E^2)
and
3-DIMENSIONAL EUCLIDEAN GEOMETRY (E^3)

How we visualize them and make copies if them

Form of the definition for $\mathbf{E}^2$ (one of several approaches):

-infinite set of basic "atomic" objects called points

-special sets of points to be called lines

Form of the definition for $\mathbf{E}^3$:

-infinite set of basic "atomic" objects called points

-special sets of points to be called lines

-special sets of points to be called planes

Definitions: A given set S of points in $\mathbf{E}^2$ is said to be collinear if there is some line L in $\mathbf{E}^2$ such that every point in S is also in L.

A given set S of points in $\mathbf{E}^3$ is said to be collinear if there is some line L in $\mathbf{E}^3$ such that every point in S is also in L. A given set S in $\mathbf{E}^3$ is said to be coplanar if there is some plane M such that every point in S is also in M.

INCIDENCE RELATIONS
(Part of definition of $\mathbf{E}^2$ and $\mathbf{E}^3$)

For $\mathbf{E}^2$: Given two distinct points, there is a unique (one and only one) line which contains both points.

For any two points P and Q, there is a measure which gives the distance d(P,Q) between P and Q.

For $\mathbf{E}^3$: Given two distinct points, there is a unique (one and only one) line which contains both points.

Given two distinct points in a plane M, the unique line determined by those points must lie entirely in (be a subset of) M.

Given three distinct non-collinear points, there is a unique plane which contains all three points. (Equivalent assertion: Given two intersecting lines, there is a unique plane which contains both lines.)

Given two planes which intersect (have a common point), then their intersection (the set of all common points) must be a line.

For any two points P and Q, there is a measure which gives the distance d(P,Q) between P and Q.

Every plane in $\mathbf{E}^3$ is a copy of $\mathbf{E}^2$.

PARALLELISM
(Definitions)

For $\mathbf{E}^2$: Two lines are said to be parallel if they do not intersect.

For $\mathbf{E}^3$: Two lines are said to be parallel if they do not intersect but do lie in a common plane.

Two lines are said to be skew if they do not intersect and do not lie in a common plane.

If each of two given distinct lines is parallel to the same third line, then those two given lines must be parallel.

CONGRUENCE

The idea of congruence is fundamental for $\mathbf{E}^2$ and $\mathbf{E}^3$. A mapping on $\mathbf{E}^2$ (or $\mathbf{E}^3$) is a function f which carries every point P to some corresponding point f(P). A mapping is said to be an isometry if for every pair of points P and Q, $d(f(P),f(Q)) = d(P,Q)$. A set of points S is said to be congruent to a set of points S′ if there is an isometry which carries S exactly onto S′.

An angle consists of two rays (half-lines) from a common vertex point. An angle is said to be a right angle if it is congruent to the "supplementary" angle formed by reversing the direction of one of its rays.

PERPENDICULARITY

For $\mathbf{E}^2$: Two lines are said to be <u>perpendicular</u>, if they intersect at a right angle.

For $\mathbf{E}^3$: Two lines are said to be <u>perpendicular</u>, if they intersect at a right angle.

A line and a plane are said to be <u>perpendicular</u>, if they intersect at a single point P and if the given line is perpendicular to every line lying in the given plane and going through P.

PROJECTIONS

For $\mathbf{E}^2$:

Fact: *For a given <u>line</u>* L *and any point* P, *there is a unique line* L′ *such that* L′ *contains* P *and is perpendicular to* L. The intersection point of L′ with L is called the <u>projection</u> of point P on L.

For any set S of points and any line L, the set of all projections on L of points in S is called the <u>projection</u> of S on L.

For $\mathbf{E}^3$:

Fact: *For a given <u>plane</u>* M *and any point* P, *there is a unique line going through* P *and perpendicular to* M. The intersection point this line with M is called the <u>projection</u> of the point P on the plane M. For any set S of points, the set of all projections on M of points in S is called the <u>projection</u> of S on M.

PROJECTIONS (continued)

Fact: *For a given line* L *and any point* P, *there is a unique plane going through* P *and perpendicular to* L. The intersection point of L with this plane is called the projection of the point P on the line L. For any set S of points, the set of all projections on L of points in S is called the projection of S on L.

MISCELLANEOUS DEFINITIONS

(to be found in Chapter 1 of the text)

parallelogram
dihedral angle
polygon
n-gon
quadrilateral
planar polygon
convex polygon
regular polygon
polyhedron
n-hedron
convex polyhedron
regular polyhedron
octahedron
dodecahedron
icosahedron
parallelepiped
prism

DEFINITIONS continued

pyramid
sphere
cylinder
cone

Also: simple mensuration formulas (area and volume)