

18.022 Hour Test

sample test 1

CLOSED BOOK: NO BOOKS, NOTES, OR CALCULATORS

Name

Rec.Instr.

Rec.Time

Please show all your work on this paper, and identify your answers clearly. Use backs of pages if necessary. Points for each question are as shown (for a total of 120 points). If you are stalled on one question, go on to the next.

1. A straight line is given by the path $\vec{R}(t) = (1+t)\hat{i} + 3t\hat{j} + (1-t)\hat{k}$.

(a) (6) Find a non-zero vector parallel to this line.

(b) (14) For what values of c and d will this line lie in the plane $x + y + cz = d$

2. (20) In Cartesian coordinates, consider the position arrows representing the vectors $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j} - \hat{k}$. Find a vector \vec{C} such that the position arrow for \vec{C} bisects the angle between the position arrows for \vec{A} and \vec{B} . (Hint. Use unit vectors. Visualize in the plane determined by \vec{A} and \vec{B} . Note that \vec{C} is not unique.)

3. (5) (a) Simplify $\vec{A} \cdot ((\vec{B} \times \vec{A}) \times \vec{A})$.

(5) (b) Simplify $\vec{A} \times ((\vec{B} \times \vec{A}) \times \vec{A})$.

(5) (c) Find $d\vec{B}/dt$ for $\vec{B} = [\vec{A}, \dot{\vec{A}}, \ddot{\vec{A}}]\vec{A}$, where \vec{A} is a vector function of t .

(5) (d) Find $d\vec{C}/dt$ for $\vec{C} = (\vec{A} \times \vec{B}) + \vec{A}$, where \vec{A} and \vec{B} are vector functions of t .

4. (20) Find the projection of the point $(1, 2, 1)$ on the line L given by the path $\vec{R}(t) = (\hat{i} - \hat{j})t + \hat{k}$. (Hint. Recall that the projection of a point P on L is the intersection of L with the unique plane which goes through P and is perpendicular ("normal") to L .)

5. Let lines L_1 and L_2 have the paths $\vec{R}_1(t) = \vec{A}_1 t + \vec{B}_1$ and $\vec{R}_2(t) = \vec{A}_2 t + \vec{B}_2$ respectively, for given vectors $\vec{A}_1, \vec{B}_1, \vec{A}_2,$ and \vec{B}_2 . We wish to determine whether or not L_1 and L_2 intersect at a unique point. We could proceed as follows:

(1) Decide whether or not L_1 and L_2 lie in a common plane.

(2) Decide whether or not L_1 and L_2 are distinct and non-parallel.

(10) (a) For what results in (1) and (2) can we conclude that L_1 and L_2 intersect at a unique point?

(10) (b) Carry out this procedure for the case where $\vec{R}_1(t) = \hat{i}t - (\hat{j} + \hat{k})$ and $\vec{R}_2(t) = \hat{j}t - \hat{i}$.

6. (20) A particle moves so that its cylindrical coordinates are given as functions of time by $r = t$, $\theta = t^2$, $z = t$, ($0 \leq t$). (a) Express velocity \vec{v} and acceleration \vec{a} at time $t = 1$ in the coordinate frame at time $t = 1$.

(b) Find the curvature at time $t = 1$.

(Recall the formulas $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$ and $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} + \ddot{z}\hat{z}$).