

18.022 Hour Test
November- 29, 2005

CLOSED BOOK; NO BOOKS, NOTES, OR CALCULATORS

Name	Rec.Instr.	Rec.Time
------	------------	----------

Please show all your work on this paper, and identify your answers clearly. Use backs of pages if necessary. Points for each question are as shown (for a total of 100 points). If you have difficulty on a problem, go on to the next.

1. (20) The directed path C in the xy plane consists of two parts: first, a left semicircle from $(0,-1)$ to $(0,1)$, with center at the origin; and then, second, a straight segment from $(0,1)$ to $(2,1)$. Let $\vec{F} = -y\hat{i} + x\hat{j}$. Find $\int_C \vec{F} \cdot d\vec{R}$. (Be careful about path-direction vs. parameter-direction.)

Answer:

SCORE

1.

2.

3.

4.

5.

TOTAL:

2. (20) Let S be the rectangular surface with vertices at $(0,0,0)$, $(1,0,0)$, $(1,2,2)$, and $(0,2,2)$. Let S be directed upward. Let $\vec{F} = y^2\hat{i} - z^2\hat{j} + x^2\hat{k}$. Find the flux of \vec{F} through S .

Answer:

3. (20) Let the surface S be the upper hemisphere of radius 1 with center at the origin. S is directed upward. Let $\vec{F} = x\hat{i} - y\hat{j} + z\hat{k}$. Find the flux of \vec{F} through S by introducing a circular disk S' in the xy plane so that S and S' form a closed surface and by then applying the divergence theorem to \vec{F} over the hemispherical solid region enclosed by S and S' .

Answer:

4. (20) Let S be the rectangle in the xy plane with vertices $(0,0,0)$, $(2,0,0)$, $(2,1,0)$, and $(0,1,0)$. S is directed up. Let $\vec{F} = z^2\hat{i} + x^2\hat{j} + y^2\hat{k}$. Let C be the boundary of S . Choose an appropriate direction for C , and verify Stokes's theorem for \vec{F} , S , and C .

Answer:

5. (20) \vec{F} is a divergenceless vector field whose domain is all of space except for the two points

$A = (0,0,0)$ and $B = (0,1,0)$. For any point P , let $S(P,a)$ be the outward directed sphere with radius a and center at P . You are given that the flux of \vec{F} through $S(A,1/2)$ is 1 and through $S(A,3/2)$ is -3.

(a) What is the flux of \vec{F} through $S(B,1/2)$?

(b) What are the possible values that can occur for the flux of \vec{F} through $S(P,a)$ as P and a vary?

Answer:

Footnote (unrelated to today's test): In the recent "practice problems", the correct answer to 8e is "T".