CARTESIAN COORDINATE SYSTEM:

Three mutually perpendicular axes are chosen:

-through a common point ("origin")

-each axis a number line with zero at origin

-right-handed labels (usually x, y, z)

-location of origin arbitrary

-directions of axes arbitrary, (subject to mutual perpendicularity)

Coordinates for any given point P in $\mathbf{E}^3$

-unique triple of real numbers obtained by taking unique projection of P on each axis

-values depend on choice of origin and of axis directions

The unit vectors $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ for a given coordinate system

The three unit vectors with positive directions of, respectively, the x-axis, y-axis, and z-axis. Note the following vector-algebra identities:

-dot product $\hat{\imath}\cdot\hat{\imath}=1, \hat{\imath}\cdot\hat{\jmath}=0, \hat{\imath}\cdot\hat{k}=0$. Similarly for $\hat{\jmath}$ and $\hat{k}$.

-cross product

$\hat{\imath}\times\hat{\imath}=\vec{0}, \hat{\imath}\times\hat{\jmath}=\hat{k}, \hat{\jmath}\times\hat{\imath}=-\hat{k}$; similarly for $\hat{\jmath}, \hat{k}$.

The $\hat{i}, \hat{j}, \hat{k}$ - representation of a given vector $\vec{A}$ for a given Cartesian system.

Represent $\vec{A}$ by an arrow with tail-point at the origin. Let (a_1, a_2, a_3) be the coordinates of the head point P. Take an arrow for a_1**i** with tail at the origin, then an arrow for a_2**j** with tail at the head of the arrow for a_1**i**, then an arrow for a_3**k** with tail at the head of the arrow for a_2**j**. By geometry, the head point of the last arrow is P. Conclusion: $\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ by the definition of vector addition.

Note that $a_1 = \hat{i} \cdot \vec{A}$, $a_2 = \hat{j} \cdot \vec{A}$, and $a_3 = \hat{k} \cdot \vec{A}$. Thus we have the identity:

$\vec{A} = (\vec{A} \cdot \hat{i})\hat{i} + (\vec{A} \cdot \hat{j})\hat{j} + (\vec{A} \cdot \hat{k})\hat{k}$. This is an example of the *frame identity*.

The coordinate formulas for our four basic vector operations now follow, by using the laws of vector algebra, as shown in the text.

A **frame in E^3** is a set of three mutually perpendicular unit vectors. Given a chosen origin point, a frame can be used to define axes for a new Cartesian coordinate system at that origin.

Triple products are special combinations of the dot and cross products which have useful properties for simplifying expressions of vector algebra. See §3.6.

TRIPLE PRODUCTS

Scalar triple product:

Has form $A\cdot(B\times C)$ or $(A\times B)\cdot C$.

Using coordinate formulas for operations, we can show that these two expressions always have the same value and that that value is a scalar. Let [ABC] abbreviate the second expression. We can also conclude: [ABC] = [BCA] = [CAB] = –[BAC] = –[ACB] = –[CBA]. Furthermore, the absolute value | [ABC]| is equal to the volume of the parallelpiped determined by taking arrows for A, B. C with a common tail point. See text fig. [6-1].

Vector triple product:

Has form $(A\times B)\times C$ or $A\times(B\times C)$. These two expressions rarely have the same value. For the second expression, we have the value $(A\cdot C)B-(A\cdot B)C$, and for the first we have $(C\cdot A)B-(C\cdot B)A$. Note that the first value is a vector lying in the plane of A and B, while the second lies in the plane of B and C

The triple products are especially useful for simplifying expressions of vector algebra. See for example, the simplifications in the text for $(A\times B)\cdot(C\times D)$ and for $(A\times B)\times(C\times D)$. (The "scalar quadruple product" and the "vector quadruple product".)