18.022 Sample Test 1
ANSWERS
CLOSED BOOK; NO BOOKS, NOTES, OR CALCULATORS

Name ____________________ Rec.Instr. ____________________ Rec.Time ____________

Please show all your work on this paper, and identify your answers clearly. Use backs of pages if necessary. Points for each question are as shown (for a total of 100 points). If you are stalled on one question, go on to the next.

1. A straight line is given by the path $\vec{R}(t) = (1+t)\hat{i} + 3t\hat{j} + (1-t)\hat{k}$.

(a) (6) Find a non-zero vector parallel to this line.

i + 3j − k

(b) (14) For what values of c and d will this line lie in the plane x + y + cz = d ?

c = 4, d = 5

2. (20) In Cartesian coordinates, consider the position arrows representing the vectors $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j} - \hat{k}$. Find a vector $\vec{C}$ such that the position arrow for $\vec{C}$ bisects the angle between the position arrows for $\vec{A}$ and $\vec{B}$. (Hint. Use unit vectors. Visualize in the plane determined by $\vec{A}$ and $\vec{B}$. Note that $\vec{C}$ is not unique.)

Let A* be unit vector of A and B* be unit vector of B. Take C = A* + B*

3. (5) (a) Simplify $\vec{A} \cdot ((\vec{B} \times \vec{A}) \times \vec{A})$. **= zero vector O**

(5) (b) Simplify $\vec{A} \times ((\vec{B} \times \vec{A}) \times \vec{A})$. **= (A dotA) (B cross A)**

(5) (c) Find $d\vec{B}/dt$ for $\vec{B} = [\vec{A}, \dot{\vec{A}}, \ddot{\vec{A}}]\vec{A}$, where $\vec{A}$ is a vector function of t.

= [A,Adot,Atriple dot]A + [A,Adot,Adoubledot]Adot

(5) (d) Find $d\vec{C}/dt$ for $\vec{C} = (\vec{A} \times \vec{B}) + \vec{A}$, where $\vec{A}$ and $\vec{B}$ are vector functions of t.

= (Adot cross B) + (A cross Bdot) + Adot

4. (20) Find the projection of the point (1,2,1) on the line L given by the path $\vec{R}(t) = (\hat{i} - \hat{j})t + \hat{k}$. (Hint. Recall that the projection of a point P on L is the intersection of L with the unique plane which goes through P and is perpendicular ("normal") to L.)

(−1/2,1/2,1)

5. Let lines L_1 and L_2 have the paths $\vec{R}_1(t) = \vec{A}_1 t + B_1$ and $\vec{R}_2(t) = \vec{A}_2 t + \vec{B}_2$ respectively, for given vectors $\vec{A}_1, \vec{B}_1, \vec{A}_2,$ and $\vec{B}_2$. We wish to determine whether or not L_1 and L_2 intersect at a unique point. We could proceed as follows:

(1) Decide whether or not L_1 and L_2 lie in a common plane.

(2) Decide whether or not L_1 and L_2 are distinct and non-parallel.

(10) (a) For what results in (1) and (2) can we conclude that L_1 and L_2 intersect at a unique point? **Lines must be coplanar, distinct, and not parallel**

(10) (b) Carry out this procedure for the case where $\vec{R}_1(t) = \hat{i}t - (\hat{j} + \hat{k})$ and $\vec{R}_2(t) = \hat{j}t - \hat{i}$. **If lines intersect, they must lie in a common plane and (A1 cross A2) = k must be normal to this plane. Plane must have equation z = c some c. But B1 ⇒ c = –1 and B2 ⇒ c = 0.**

6. (20) A particle moves so that its cylindrical coordinates are given as functions of time by $r = t$, $\theta = t^2$, $z = t$, $(0 \leq t)$. (a) Express velocity $\vec{v}$ and acceleration $\vec{a}$ at time $t = 1$ in the coordinate frame at time $t = 1$. **Use r = t = 1, r dot = 1, r double dot = 0, θ = t² = 1, θ dot = 2t = 2, θ double dot = 2, z = t = 1, z dot = 1, and z double dot = 0, getting v = r hat + 2 θ hat + z hat and a = –4 r hat + 6 θ hat.**

(b) Find the curvature at time $t = 1$. **Use κ = |v × a| / |v|³ with frame {r hat, θ hat, z hat}, getting v × a = –6 r hat –10 θ hat + 14 z hat and κ = sq root (83/54).**

(Recall the formulas $\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{z}$ and $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} + \ddot{z}\hat{z}$).