CYLINDRICAL COORDINATES; DEFINING EQUATIONS

COORDINATE SURFACES, COORDINATE CURVES, COORDINATE VECTORS, AND COORDINATE FRAMES

PATHS, VELOCITY, AND ACCELERATION IN CYLINDRICAL COORDINATES

CENTRAL-FORCE MOTION, ANGULAR MOMENTUM, AND KEPLER'S SECOND LAW

Cyl. Coords and Defining Equations: See figure [1-1] for geometric relationship between the cyl. coordinates $\{r,\theta,z\}$ and their "associated" Cartesian system of $\{x,y,z\}$. Note that for any given point P, the usual position vector $\mathbf{R_P}$ in Cartesian coordinates is related to the associated cyl coordinates by the vector equation:

$$\mathbf{R_P}(x,y,z) = r\cos\theta\,\mathbf{i} + r\sin\theta\,\mathbf{j} + z\,\mathbf{k}.$$

This vector equation can also be expressed as three scalar equations, see (2.1).

Coordinate Surfaces and Coordinates Curves: Given a point P, we can hold the values fixed for any chosen pair of cyl coordinates and let the third cyl coordinate vary. This gives us the *coordinate curve* through P for that third coordinate. Thus the *r-curve* through P is a ray which starts on the z-axis and runs through P parallel to the plane $z = 0$; if we use the defining equation with r as parameter and with the fixed values for θ and z given by P, all the concepts and definitions of Chapter 6 can all be applied to the r-curve through P. Similarly, the *θ-curve* through P is a circle of radius r with center on the z-axis and in a plane perpendicular to that axis; it uses θ as parameter. Similarly, the *z-curve* through P is a line through P parallel to the z-axis, and can use z as parameter.

Unit coordinate vectors. For any point P not on the z-axis, the unit tangent vector at P to the r-curve through P is called the unit vector $\mathbf{r_P}$, or simply, the unit vector $\mathbf{r}$ at P. The unit vectors $\boldsymbol{\theta}_\mathbf{P}$ and $\mathbf{z_P}$ are defined similarly. For any point P not on the z-axis, these three unit vectors at P are mutually orthogonal and form the *coordinate frame* at P. For any point P, if we use the frame identity, the components of any given vector with respect to the coordinate frame at P can be found. It is important to note that the directions of the unit coordinate vectors $\mathbf{r}$ and $\boldsymbol{\theta}$ may change as we shift from one given point P to another.

Paths. In certain problems, the *path* **R**(t) of a given "moving point" P may have a simpler expression in cyl coordinates than in Cartesian coordinates. In such cases we view the position of P as given by the dependence of its cyl coordinates upon our parameter t. Thus we will begin with a parametric system of the form r = r(t), θ = θ(t), and z = (t). Then, using the cyl coordinate frame, we can express **R**(t) as **R**(t) = r**r** + z**z**, where **r** and **z** are unit coordinate vectors and r and z are the given scalar functions r(t) and z(t), see figure [5-1]. We must immediately note however, that the unit vector **r** will change in direction as our point P(t) moves along the path. This means that **r** must itself be treated as a vector-valued function of t. The dependence of **r** on t can be expressed in the associated Cartesian system by:

r(t) = r cos θ **i** + r sin θ **j** , where r and θ are the given scalar functions r(t) and θ(t). Similarly we have:

θ(t) = − r sin θ **i** + r cos θ **j**.

Velocity and acceleration. We can now proceed to apply the methods of calculus to find expressions for *velocity* and *acceleration* in terms of the cyl frame and the functions r(t). θ(t), and z(t). This development is derived in the text, beginning at the bottom of page 149. We can then use these results, for example, to find the curvature at any point on a curve which has been presented in cyl coordinates.