18.022 Lecture notes

The course is divided into 6 parts:

Part 1 (Lectures I – VII): Euclidean Spaces and Vector Algebra
Part 2 (VIII – XIII): Differential Calculas for Scalar Fields and Functions of Several Real Variables.
Part 3 (XIV – XVII): Multiple Integrals
Part 4 (XVIII – XXII): Line Integrals and Surface Integrals in Scalar and Vector Fields
Part 5 (XXIII – XXVIII): Vector Integral Calculus in Two and Three Dimensions
Part 6 (XXIX – XXXV): Linear Algebra in Multivariable Calculus

Lecture I
The Three-dimensional Space

We refer to the Euclidean two- and three-dimensional spaces as $\mathbf{E}^2$ and $\mathbf{E}^3$, respectively. Euclidean spaces have a measure of *distance* between points; for every two points P and Q, we denote it by d(P, Q). This measure satisfies the following two laws:

i) For any two given points P and Q, d(P, Q) = 0 if and only if P = Q.
ii) For any three given points P, Q and R, d (P, R)$\leq$d (P,Q) + d (Q, R).

The following are some elementary facts in $\mathbf{E}^3$

1) Given two distinct points P and Q, there is a unique line that contains both P and Q.
2) Given three non-collinear (not all contained by a single line) points P, Q, and R, there is a unique plane that contains P, Q and R.
3) Given two distinct points P and Q on a plane M, then M contains the line determined by P and Q.
4) Given two intersecting planes, their intersection is a line.

Any *isometry* is a mapping f from the points in $\mathbf{E}^3$ onto the points in $\mathbf{E}^3$, such that, for every pair of points P and Q, d(f(P), f(Q)) = d(P, Q). Two subsets of $\mathbf{E}^3$ (subsets), are said to *congruent* if there is an isometry which carries one figure onto the other.

Parallelism and Perpendicularity

Two lines are said to be *parallel* if they lie in common plane and they do not intersect.
Two lines are said to be *skew* if they do not lie in a common plane
Two planes that do not intersect are said to be *parallel*.
A line and a plane that do not intersect are said to be *parallel*

(1) For any line L and any point P not on L, there exists a unique line through P parallel to L.
(2) For any plane M and any point P not on M, there exists a unique plane through P parallel to M.

An *angle* is a figure formed by two rays (half-lines) with common vertex (end-point). Two lines are *perpendicular* if they intersect and form right angles. A line L and a plane M are *perpendicular* if they intersect at a single point P and L is perpendicular to every line lying in M and going through P.

Projections

In $\mathbf{E}^2$, for a given line L and any point P, there exists a unique line L' through P perpendicular to L. The intersection point of L' and L is called the *projection* of point P on L. For any set S of points and any line L, the set of all projections on L of points in S is called the *projection* of S on L.

In $\mathbf{E}^3$, for a given plane M and any point P, there exists a unique line going through P and perpendicular to M. The intersection point of this line and M is called the *projection* of the point P on the plane M. For any set S of points, the set of all projections on M of points in S is called the *projection* of S on M.

For a given line L and any point P, there is a unique plane going through P and perpendicular to L. The intersection point of L with this plane is called the projection of the point P on the line L. For any set S of points, the set of all projections on L of points in S is called the projection of S on L.

For more definitions and elementary facts in $\mathbf{E}^2$ and $\mathbf{E}^3$, read the second part of Chapter 1 of the textbook.