18.024 Homework 3 - Solutions

Problem 1.
If (n_x, n_y, n_z) is the normal vector to a plane M, it can be shown that

$$n_x \cdot x + n_y \cdot y + n_z \cdot z = c$$

is a cartesian equation for the plane M. Then, three planes in V_3 intersect iff the system of three equations and three unknowns:

$$\begin{aligned} n_1x \cdot x + n_1y \cdot y + n_1z \cdot z &= c_1 \\ n_2x \cdot x + n_2y \cdot y + n_2z \cdot z &= c_2 \\ n_3x \cdot x + n_3y \cdot y + n_3z \cdot z &= c_3 \end{aligned}$$

has a unique solutions. This system has a unique solution if the vectors (n_1x, n_1y, n_1z), (n_2x, n_2y, n_2z), (n_3x, n_3y, n_3z) are linear independent. These vectors are the normal vectors, so the planes intersect in a single point iff the normal vectors are linear independent.

Problem 2.

$$det(A) = 5$$

$$inv(A) = \frac{1}{5}\begin{pmatrix} 4 & 3 & 2 & 1 \\ 3 & 6 & 4 & 2 \\ 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

Problem 3. P492: 3.

$$V = (\hat{i} + \hat{j}) \cdot \Big((\hat{j} + \hat{k}) \times (\hat{k} + \hat{i}) \Big) = 2$$