

## 18.024 Homework 6 - Solutions

### Problem 1.

Since  $S$  is bounded, there exists a rectangle  $Q$  such that  $S \subset Q$ . Let

$$g(x) = \begin{cases} 1 & \text{if } x \in S \\ 0 & \text{if } x \in Q - S \end{cases}$$

It is easy to prove that  $g$  is continuous on all of  $Q$  except on  $\partial S$ . Since  $\int \int_S f = \int \int_Q g$ , one exists iff the other one exists. Therefore,  $\int \int_Q g$  exists, so the set of points where  $g$  is discontinuous has to have zero content. Therefore,  $\partial S$  has zero content.

### Problem 2.

$$V = \int_1^3 \int_{-x}^x (x^2 - y^2) dx dy = \frac{80}{3}$$

### Problem 3.

$f$  is continuous in all of  $Q$ , except for the line  $x = \frac{1}{2}$ . This line can be covered by a rectangle of  $\epsilon \times 1$ , so it has content zero. Therefore, the integral of  $f$  over  $Q$  exists.

The line integral  $\int_0^1 f(\frac{1}{2}, y) dy$  doesn't exist because  $\underline{I} = 0$  and  $\bar{I} = 1$ .