18.024 Homework 2 - Solutions

Problem 1. P. 483: 14.

Let $L = (Q, A) = \{Q + aA\}$. Note that $M = (Q, A, P - Q) = \{Q + sA + t(P - Q)\}$ contains P (letting $s = 0, t = 1$) and contains every point of L (if $X = Q + aA$, let $s = a, t = 0$).

Now, take P, Q and $T \in L$. They all belong to the plane that includes L and P. By theorem 13.10 they define a unique plane. SoM is the only plane that satisfies the conditions.

Problem 2. P. 604: 12.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2 + bc & b(a + d) \\ c(a + d) & bc + d^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Then we have 4 equations. $a = \pm d$. If $a = d$, we get that $b = c = 0$ and $a = d = \pm 1$. If $a = -d$, b and c are arbitrary and a must satisfy $a^2 = 1 - bc$. Hence the solutions are: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} a & b \\ c & -d \end{pmatrix}$ with $a^2 = 1 - bc$.

Problem 3.

Since the rows of the matrix A form a basis for V_n, $dim(A) = n$. So the reduced echelon form will have n pivots, and the diagonal. Each of the rows will have just one non-zero entry (in the diagonal). In the reduced echelon form, the pivots are equal to 1, so the reduced echelon form of A is I_n.