18.024 Homework 6 - Solutions

Problem 1.

Solutions at the back of the book. Note that part c) was easier to solve using the chain rule.

Problem 2.

Want to minimize the function $A(x, y, z) = 2(xy + yz + zx)$ with the constraint that $V = xyz = 10$. Then, we want to minimize $f(x, y) = xy + \frac{10}{x} + \frac{10}{y}$.

$$\frac{\partial f}{\partial x} = y - \frac{10}{x^2} = 0 \rightarrow y = \frac{10}{x^2}$$

$$\frac{\partial f}{\partial y} = x - \frac{10}{y^2} = 0 \rightarrow x = \frac{10}{y^2}$$

Solving we get that $x = y = z = 10^{\frac{1}{3}}$. This is the only stationary point of the function with the constraint. Simple checking gives that it is a relative minimum. Since there are no other stationary points, it is actually an absolute minimum. Then, the rectangular box that minimizes the surface area is the cube.

Problem 3.

Absolute minimum at (1,1).
Absolute maximum at $(\frac{1}{2}, \frac{1}{2})$.
Saddle points at (0,0), (0,1), (1,).