18.024 Homework 5 - Solutions

Problem 1.

a) Note that $f'(\mathbf{a},\mathbf{y}) = -f'(\mathbf{a},-\mathbf{y})$. So if one is positive the other one is negative, and $f'(\mathbf{a},\mathbf{y})$ can't be positive for every $\mathbf{y}$.

b)For a given $\mathbf{y}$, let $f(\mathbf{x}) = \mathbf{x}\cdot\mathbf{y}$.
Then, $f'(\mathbf{x},\mathbf{y}) = \mathbf{y}\cdot\mathbf{y} > 0$.

Problem 2.

a)$\nabla r = \frac{\vec{r}}{\|\vec{r}\|}$, so it it the unit vector in the direction of $\vec{r}$.

b) One way is by using the hint and using induction. It can also be shown straightforward:

$$\frac{\partial(r^n)}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2+z^2)^{\frac{n}{2}} = nx(x^2+y^2+z^2)^{\frac{n-2}{2}} = nxr^{n-2}$$

Then,

$$\nabla(r^n) = nr^{n-2}\vec{(r)}$$

c) The method used above is valid for any n, in particular is valid for $n = 0$ or n negative.

d) It is easy to show that $f(x,y,z) = \frac{1}{2}r^2$ works.

Problem 3.

The tangent planes to spheres are perpendicular if and only if the point of intersection and the centers of the spheres form a rectangular triangle, from where the answer $c = \pm\sqrt{3}$ follows.