18.034 Problem Set #3 due Th. 03/01/07 by noon,

Problem 1. pp. 38; #9.

Problem 2. pp. 38; #4(d), #5 for (d), #6 for (d).

Problem 3. Euler's equi-dimensional equation is the DE

$$x^2u'' + pxu' + qu - 0; \quad p, q \text{ are constants.}$$

(a) Show that the setting $x = e^t$ changes the DE into an equation with constant coefficients.
(b) Use this to find the general solution to $x^2u'' + xu' + u = 0$.
(c) For which values of p, the general solutions of the DE $x^2u'' + pxu' + 2u = 0$ are defined on the entire real axis $(-\infty, \infty)$?

Problem 4. The DE $y' = f(y)$, where

$$f(y) = \begin{cases} y^2 \sin(\frac{1}{y}) & \text{for} \quad y \neq 0 \\ 0 & \text{for} \quad y = 0, \end{cases}$$

has a constant solution $y = 0$. Is this critical point (a) stable? (b) strictly stable?

Problem 5. pp. 46; #7(b), pp. 46; #5.

Problem 6. Let f be a solution of the DE $u'' + q(x)u = 0$, where q is real valued, such that f and f' are both bounded for all $x \in \mathbb{R}$. Prove that if g is a second, linearly independent solution of the DE then $\lim_{x\to\infty} g(x) = 0$ cannot happen.

Problem 7. pp. 54; #5.