18.034 Midterm #2, Sample TF Questions
The exam is Wed. 04/18/07, 1:00-1:55pm,

1. Consider the differential equation $y^n + a_1 y^{n-1} + \cdots a_n y = e^{-x}$. If $a_1, a_2, \ldots, a_n$ are all positive, then every solution tend to zero as $x \to \infty$.

2. Let ϕ_1, ϕ_2, ϕ_3 be three solution of $y''' + ay'' + by' + cy = 0$. If $W(\phi_1, \phi_2, \phi_3) = 2e^{3x}$, then every solution is expressed uniquely as $c_1\phi_1(x) + c_2\phi_2(x) + c_3\phi_3(x)$.

3. $\mathcal{L}[e^{t^2}](s)$ exists for $s > 0$.

4. $\mathcal{L}[t^k](s)$ exists for all k.

5. Let $y(t)$ be the solution of the initial value problem $y'' + 2y' + 2y = u_0(t)$ with $y(0) = y'(0) = 0$. (Here, $u_0(t)$ is the unit step function.) Then, $\mathcal{L}[y](s) = 1/(s^2 + 2s + s)$.

6. $f(x, y) = x^2|y|$ satisfies a Lipschitz condition on the rectangle $|x| \le 1$, $|y| \le 1$.

7. $f(x, y) = xy^2$ satisfies a Lipshcitz condition on the strip $|x| \le 1$, $|y| < \infty$.

8. $f(\mathbf{x}) = |\mathbf{x}|$ satisfies a Lipschitz condition on $|\mathbf{x}| \le 1$.

9. If $f(x, y)$ satisfies a Lipschitz condition on a closed bounded domain, then so does $f^2(x, y)$.

10. Given $L[y] = y^{(n)} + a_1 y^{(n-1)} + \cdots + a_n y$ such that $|a_j| \le b_j$ for $j = 1, 2, \ldots, n$, let ϕ is a solution of $L[y] = 0$ on $t \in [0, 1]$. Then, $|\phi(x)| \le |\phi(0)|e^{(1+b_1+b_2+\cdots+b_n)}$.

11. $f * 1 = f$.

12. $f * g = g * f$.

13. The system of differential equations

$$\frac{dx}{dt} = ax + by, \quad \frac{dy}{dt} = cx + dy$$

has only one critical point.

14. The system

$$\frac{dx}{dt} = 4x - y, \quad \frac{dy}{dt} = 2x + y$$

has two solutions $x = e^{3t}, y = e^{3t}$ and $x = e^{2t}, y = e^t$.

15. If $|f(t, x)| \le K$ for all $-\infty < t, x < \infty$ then the solution $x(t)$ of the initial value problem $x' = f(t, x)$, $x(0) = 0$ exists for all $-\infty < t < \infty$.

16. The sequence of Picard approximations of the initial value problem $x' = t^2 + x^2$, $x(0) = 0$ converges for all t.

17. The sequence of Picard approximations of the initial value problem $x' = x(1 - 2t)$, $x(0) = 1$ converges for all t.

18. $x(t) = 1$ is the only solution of the initial value problem $x' = t\sqrt{1 - x^2}$, $x(0) = 1$.

19. $x(t) = -1$ is the only solution of the initial value problem $x' = t(1 + x)$, $x(0) = -1$.

20. If $F(t, x)$ and $\partial F/\partial x$ are continuous in the rectangle $|t| \le 1$, $|x| \le 1$, then two solutions $y' = F(t, y)$ and $z' = F(t, z)$ satisfy $|y(t) - z(t)| \le |y(0) - z(0)|e^{M|t|}$ for some $M > 0$.

Answers.
FTFFFTFFTT
FTFFTFTFTT