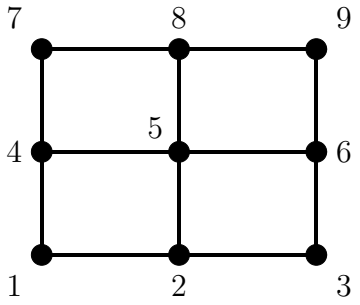
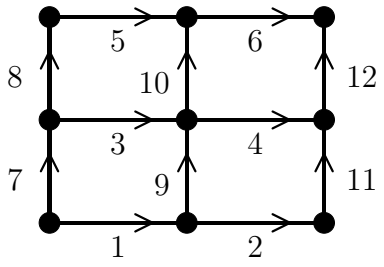
18.085 Quiz 1 October 12, 2005 Professor Strang

Your PRINTED name is: _____ **Grading** **1**
2
3

1) **(40 pts.)** Here is a network with 9 numbered nodes and then separately the 12 edges.





(a) What is the shape of the incidence matrix A? What is its 4th row?

(b) What is the 5th column of A? What is the $(5,5)$ entry in $A^{\mathrm{T}}A$? Write the whole 5th row of $A^{\mathrm{T}}A$.

(c) How many independent solutions to Kirchhoff's Law $A^{\mathrm{T}}w = 0$? *Find one of them.*

(d) Without writing down this matrix A, explain why $A^{\mathrm{T}}A$ is or is not symmetric positive definite.

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2) **(30 pts.)** (a) For a hanging elastic bar, with $u(0) = 0$ at the top and $u'(0) = 0$ at the bottom and elastic constant $c(x) = 1$, what is the displacement $u(x)$ when a *unit point load* $f(x) = \delta(x - a)$ acts at the point $x = a$? Draw a graph of $u(x)$.

(b) What is the limit of $u(x)$ as the unit load moves to the bottom $(a \to 1)$? Suppose it moves to the top $(a \to 0)$? Draw graphs of $u(x)$ in those two cases.

(c) Choose a matrix equation that approximates the differential equation in part (a). (Describe the matrix—OK to put the load at a meshpoint.) If the load moves to the lowest meshpoint (number N), what displacements correspond to your answer in part (b)?

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3) **(30 pts.)** Suppose you measure your initial position $u_1 = b_1$, and then you measure the step lengths $u_2 - u_1 = b_2$ and $u_3 - u_2 = b_3$. At the end you make a last measurement $u_3 = b_4$.

(a) Under what conditions on b_1, b_2, b_3, b_4 will these four equations have an exact solution? Create (don't solve) a set of equations for the best estimates $\widehat{u}_1, \widehat{u}_2, \widehat{u}_3$.

(b) Draw a picture of masses, springs, and forces (write in all constants) that would lead to the same equations for the displacements.

(c) Suppose the variances for errors in the measurements are $\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2$. What equations should you solve (DON'T DO IT) for the statistically best estimate $\widehat{u}$? If $\sigma_4 \to \infty$ so that b_4 becomes completely unreliable, what answer do you expect for the best $\widehat{u}$?

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